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APPLICATION OF AN AGGREGATE PROGRAMMING MODEL IN FIVE-YEAR PLANNING

The article expounds a linear programming model by means of which calculations have been worked out in order to establish the 1970 plan targets. The model, broken down to 18 sectors, surveys the national economy in a very aggregated form. The aim was to produce a great number of plan variants, each of them embodying a real plan concept differing from one another though, from the aspect of general politico-economic aims and some fundamental foreign trade suppositions. The purpose of the series of calculations was to compare and analyse the different variants and to hit upon the sensitive and relatively stable points of the programmes.

In Hungary, as in most socialist countries, the two methods of constructing planning models — i.e. input-output analysis and mathematical programming — have developed in recent years more or less independently of one another. The idea of joining the two methods has repeatedly been put forward in the literature on the subject. In Hungary, too, some efforts were made to this end.* The present article gives an account of an experiment of this type, the construction of a relatively aggregate economy-wide programming model based on some 80 equations, the data material of which had been taken over from the input-output table of the National Planning Office.

The relationship between the model in question and that of "two-level planning", with its detailed and highly disaggregate mathematical programming, will be dealt with at the end of this article. This will be preceded by the description of the aggregate model itself and the analysis of some experiences gained in the course of the calculations. It is beyond the scope of this article to analyse the inferences that may be drawn from our calculations for economic policy and practical planning. The main purpose is to expound the economic and planning-methodological conclusions which lend themselves to generalization.**

* See e.g. [1].

** The numerical results of the research work and the conclusions to be derived from it for economic policy are presented in detail in a paper prepared by the authors [2].

Description of the model

In constructing the model we started from the given endowments of the country. The Hungarian economy is highly "extrovert". On the one hand, for a great number of basic raw materials we have to fall back on imports. On the other hand, as a small country we cannot undertake to develop every branch of manufacturing industry and have to aim at a reasonable division of labour with other countries. As a consequence, every investment problem in Hungary is inseparably connected with questions of foreign trade: the development projects of a sector always "compete" with the alternative of meeting the additional requirements by imports and it will always be expedient to take into account the export aspects when planning the extent of development. How to enter most advantageously into the international division of labour, is a question underlying almost every planning problem. Moreover, the country is now faced with considerable difficulties in its foreign trade and these bring the foreign-trading aspects of plan decisions even more to the fore. It was our endeavour to construct the model in a manner that the results of the computations provide an answer to questions of this type.

As regards its mathematical form, the model represents a standard problem of linear programming. The program yields the production and foreign-trade estimates for 1970, the last year of the five-year plan period.

The national economy was divided into 18 productive sectors and the majority of the model's variables is linked with this division.

The variables are of the following types:

1. Production of the i-th sector, with the capacities existing already at the beginning of the plan period;

2. Production of the i-th sector, with the additional capacities brought into being in the course of the plan period;

3. Exports of products of the i-th sector to socialist markets;

4. Exports of products of the i-th sector to capitalist markets;

5. Competitive imports from socialist markets to replace the products of the i-th sector;

6. Competitive imports from capitalist markets to replace the products of the i-th sector.*

In addition to the above, a further seventh type of variables also figured in the model, that of the so-called indicator variables.

7.1. The volume of additional consumption attainable in 1970, over and above the private consumption compulsorily prescribed in the model. The pattern of this additional private consumption has been fixed in a breakdown

* In actual practice, not all of the six types of variables could be interpreted for each sector. The number of the variables representing the economic activities is therefore less than 6 times 18.

by 18 sectors, i.e. it has been prescribed that out of 1 million forints of additional private consumption 141 thousand forints should be supplied by the food industry, 219 thousand forints by the textile industry, and so forth.

7.2. The 1970 balance of commodity trade with socialist countries.

7.3. The 1970 balance of commodity trade with capitalist countries.

As can be seen, each of the three indicator variables represents a synthetic index of the economy. From the methodological point of view, there would have been no obstacle to building into the model further indicator variables. Such could have been e.g. total domestic final demand in 1970, or additional investments over and above a compulsory minimum level, etc.

Before proceeding to describe the constraints of the model, a preliminary remark must be made on the treatment of imports. The imports of the national economy have been divided into two main classes, those of competitive and non-competitive imports. Non-competitive imports are those for which no domestically produced substitute will be available by 1970. (E.g. for the lack of certain physical factors, or for technical reasons, or because of obligations under definitely concluded trade agreements, etc.) The non-competitive import requirements of final demand were considered as constants and disregarded in the computations. The non-competitive import requirements of production were treated as a function of the production variables; for each production variable, the coefficients of non-competitive import requirements were determined both for imports from socialist and from capitalist markets. By deducting from the material requirement coefficients (i.e. from the standard technological coefficients of the input-output tables) the coefficients of non-competitive import requirements, we obtain the competitive input coefficients (which will be denoted in a later equation g_{ij}). These represent the material requirements which may be met either out of domestic production or out of imports — the program has a free choice in this respect. The competitive imports themselves, which serve to meet competitive requirements, are represented in the model by separate variables, as will be clear from the list of variables above.

After these preliminary remarks on imports, let us proceed to describe the constraints of the model.

I. Product balances. Their constants differ from the usual ones in the special treatment of imports. The structure of the product balance referring to the i-th sector will be as follows:

$$x_{i1} + x_{i2} + \overset{(S)}{y_i} + \overset{(K)}{y_i} - \overset{(S)}{z_i} - \overset{(K)}{z_i} - \qquad (1)$$

$$- \sum_{j=1}^{18} g_{ij} \, (x_{j1} + x_{j2}) - h_i w_i \geq d_i,$$

where

x_{i1}, x_{i2} = production of the i-th sector with (1) the capacities existing already before the plan period and (2) with the additional capacities brought into being in the course of the plan period,

$y_i^{(S)}$, $y_i^{(K)}$ = competitive imports to replace the products of the i-th sector, from socialist and capitalist markets, respectively,

$z_i^{(S)}$, $z_i^{(K)}$ = exports of the products of the i-th sector, to socialist and capitalist markets, respectively,

g_{ij} = competitive input coefficient (for its interpretation see the explanations concerning the treatment of imports above),

h_i = contribution of the i-th sector to one unit of additional consumption,

w_l = the indicator variable of additional consumption,

d_i = domestic final demand in 1970: private and public consumption, the increase in stocks, and the investment and renewal activities in 1970.

As can be seen, constraint (1) allocates only the products whose sources are domestic production and competitive imports, in competition with one another.

II. Resource bounds. These comprise the following constraints:

II.1. The fixed capital requirements of productive activities in 1970 must not exceed the total fixed capital available for 1970.

II.2. The total manpower requirements of productive activities in 1970 must not exceed the available manpower. Within the bounds of total manpower separate constraints are set to bound agricultural and non-agricultural labour requirements as well as the demand for male labour which constitutes at present one of the principal bottlenecks in domestic labour supply.

II.3. The import machine requirements needed to bring into being the new capacities to be created in the five-year plan period must not exceed the import machine quotas available in that period.

II.4. In the sectors which require natural resources, an upper bound is set by the limited quantity of the necessary natural resource.

III. Capacity constraints. These are the upper bounds of the variables representing production carried out with old capacities which were already in existence at the beginning of the plan period.

IV. Export and import constraints.

IV.1. Upper bounds were set to all export activities in order to express the foreign buyers' limited propensity to buy.

IV.2. Upper bounds were set to all imports from socialist markets, generally in order to express the sellers' limited propensity to sell. In some cases this constraint represents the upper bound of our own propensity to buy, as in some branches the demand can — because of definite technical, qualitative and other requirements — only partially be met by imports from socialist markets. Imports from capitalist markets are not bounded from above because purchases are generally not limited on the part of the sellers. It is only on the part of ourselves, the buyers, that upper bounds exist for foreign-exchange reasons; these, however, should be expressed not in the form of import bounds but in the form of commodity trade balances securing a definite surplus (or deficit).

IV.3. The export transactions already contracted are compulsorily prescribed in the form of lower bounds. As will be explained in detail below, the model was used to carry out not a single computation but a whole series of calculations. In some calculations the lower bounds of the type IV.3. were prescribed, in others not.*

V. Foreign exchange balances. Separate foreign exchange balances (or, to put it more precisely, balances of foreign exchange returns from and outlays on trade) have been prescribed for the socialist markets in terms of roubles and for the capitalist markets in terms of dollars. These balances contain the items originating in exports as positive ones and the outlays originating in the competitive import variables as well as in the non-competitive import requirements of the production variables as negative items.

After the survey of the system of constraints, let us briefly examine also the objective functions. Our calculations employed mainly parametric objective functions, in the following form:

$$\lambda w_i + (1-\lambda)\, w_j \rightarrow \max ! \qquad 0 \leq \lambda \leq 1. \tag{2}$$

where

λ = the parameter
w_i, w_j = two of the three indicator variables.

Let us take, for example, the following case: let w_i denote the additional consumption and w_i the balance of capitalist foreign exchange. If now the value of parameter λ is 1, the objective is "solely" to maximize consumption; if the value of the parameter is 0, the objective is "solely" to maximize the positive foreign exchange balance. With intermediate parameter values the two objectives can be combined with varying weights. In the calculations we let the parameter run over the whole interval [0, 1]. In addition, separate calculations were carried out by combining the optimization of consumption and the balance of socialist foreign exchange as well as by combining the optimization of both kinds of foreign exchange balances.

* In actual practice it was not necessary to fix the lower bounds of exports as separate constraints; they could be built into product balance (1). It contained in these cases, in addition to final domestice use, also the "compulsory" exports, with the export variables representing not the total exports but only the additional exports above the compulsory level.

The sources of the data

In working out the plan for 1970, the main source of data for the model was the input-output table constructed by the Department for Long-term Plans in the National Planning Office.

The drawing up of the plan started on the basis of traditional, non-mathematical, methods. In the first draft plan thus drawn up it was, however, not possible fully to coordinate the individual estimates; there appeared in it disproportions and potential future equilibrium disturbances. Construction of the input-output table and its discussion with practical planners helped to reveal the disproportions and potential equilibrium disturbances, and to improve the plan coordination. Finally, an input-output table was obtained which, based on the ideas of practical planners, approximately reflected the main estimates of the 1970 plan worked out in the National Planning Office.*

It is from the National Planning Office's input-output table for 1970 that the basic data of our aggregate programming model were taken, first of all the technological coefficients which served as a starting point for the estimation of the g_{ij} coefficients figuring in equation [1].

In connection with the construction of the input-output tables, the Department for Long-term Plans has also prepared so-called chessboard-type import tables which present the import structure of the individual sectors broken down into imports from socialist and capitalist markets. Here, the import requirement coefficients figured globally; later — with the help of planners in the individual sectors — they were disintegrated, in accordance with the structure of our modell into competitive and non-competitive ones.

The estimates of final domestic use were taken over from the 1970 input-output table mentioned above, which corresponded to the official plan estimates worked out on the basis of the traditional methods. The resource constraints were taken over from the official plan, whereas the demands on fixed-capital, imported machinery and manpower were derived from the plan indices based on the traditional methods considering them, as it were, as complementary data to the 1970 input-output table.

The material and import coefficients as well as the fixed-capital and manpower coefficients have been compared with the corresponding data of statistical input-output tables of a similar breakdown and where necessary, corrections were made.

The foreign trade prices were based on the Hungarian foreign trade statistics reflecting the actual transactions. It would have been expedient, instead of making direct use of the statistical figures, to work out estimates for the expected 1970 prices. However, this exceeded the possibilities of this first

* The work in the Planning Office aimed at drawing up the input-output table for 1970 was directed by Mrs. L. Újlaki, co-author of this article.

experimental calculation. The estimates of the export and import bounds were worked out in cooperation with the foreign trade experts of the National Planning Office.

From what has been said it should be clear that our data — as far as they are not based on the mere adaptation of statistical figures — were either taken over from the documents of traditional, non-mathematical, planning or based on the estimates of the practical planners in the National Planning Office. The computations carried out by means of the mathematical programming model can thus not be considered a research project independent of the work in the National Planning Office. If the calculations carried out by means of the model do differ from certain targets of the plan compiled on the basis of the traditional method, this should not be taken as a criticism of the Planning Office's work from outside but rather as a kind of "self-criticism". It is a well-known fact that the numerical results yielded by a mathematical model will depend to a high degree on the initial data fed into the electronic computer. In our case, the source of the initial data was without an exception, in one form or another, the practical planning apparatus. This close interrelation between the work carried out by means of the mathematical model and the traditional planning activities is all the more remarkable since — according to our knowledge — in a number of countries the mathematical economists can hardly come into contact with practical planning work and get little assistance from the planners.

To avoid any misunderstanding, it should be pointed out that the close connections with the Planning Office's work do not mean that the targets of the 1966/70 plan, as they were finally adopted, were based on the computations carried out by means of the aggregate model. Due to the low number of the research workers taking part in the project and to computation-technical difficulties, it was not before a rather late stage of planning that the work of aggregate programming reached completion, too late for the results obtained to have a decisive influence on the plan. Moreover, in view of the experimental character of the work and the numerous uncertainties contained in the data, we accepted the numerical results with due reservations ourselves. Under the circumstances, the mathematical programs provide rather a basis for minor partial modifications of the plan and for corrections to be carried out later on, in the course of its implementation. In the future, when drawing up the subsequent five-year plans, it should be endeavoured to be able to draw on the results of aggregate programming projects of this type already at the beginning of the planning work, when working out the initial figures.

The computation series carried out with the model

Before the introduction of the mathematical methods no simultaneous plan variants were generally prepared in Hungary. The aim of the planners was to produce a single acceptable plan proposal. With the traditional "handicraft" techniques even the compilation of this single plan proposal required such enormous efforts that there was no capacity left to work out parallelly further variants. What is more, the planners usually did not succeed to ensure coordination in all details or the complete equilibrium even of this single plan.

In the past this situation was rarely recognized as a regrettable but inevitable consequence of the use of "handicraft" techniques in planning; instead, it was tried to give it an "ideological" foundation. Among the planners and the theoretical economists engaged in planning the idea was prevailing that there existed but one "true" plan which would "express the objective laws of the economy". Occasionally, when working out their proposals and giving them a definite form, the planners would even make themselves believe that they succeeded in finding this only "true and inevitable" plan.

With the appearance of the mathematical techniques of planning it became clear that in each given concrete objective situation it is possible to work out several plans, all realistic and realizable in themselves, which differ from each other only in the extent to which they serve the various possible objectives of economic policy and in their efficiency. It is from among these that the plan to be actually carried out will have to be selected. Even the simple input-output models enable to draw up simultaneous plan variants of this type.

However, with the appearance of the mathematical programming and optimization models, the idea of the only "true" plan obtained new "ideological" foundations. True, those versed in mathematical programming will know only too well that if there is at all a feasible program satisfying all constraints, then there is usually an infinity of such programs. But with an extremely wide category of programming models — a category which includes also the linear programming models — there is (if one disregards some exceptional cases of degeneration) only one program where the objective function to be maximized will actually assume its maximum value. An only "true" optimum plan would thus exist and it is the task of the mathematical planners to find it.

This could even be true if there existed some unique objective function expressing the interests of society in a synthetic manner. We do, however, not believe in the existence of such an objective function, and consider therefore the optimality of each "optimum" program (optimum in the sense that it ensures the extreme value of the objective function) to be only relative. This program will be the relatively most advantageous — with the given numerical value of the constraints and the optimality criterion inevitably selected with a certain degree of arbitrariness. However, both the numerical values of the constraints and the selection of the objective function express not only the objective potentialities and inevitable tendencies of the economy but also the decisions of economic policy as well as the inevitably inaccurate estimates and assumptions concerning uncertain future phenomena.

In our view it is not a single „optimum" plan that should be worked out with the aid of a mathematical programming model but a whole series of plan variants. For example, on the basis of our aggregate model 43 variants have been calculated. Each of them describes a complete five-year plan and satisfies some basic proportions in production, foreign trade and consumption. The differences between the variants may be summed up as follows.

1. What are the main endeavours on which the efforts of the economy should be centred, the maximization of consumption or the improvement of the country's foreign trading position? We are aware of the fact that in long-term — 15- or 20-year — planning the improvement of any of the foreign exchange balances could not figure as an objective function as this is, in fact, not an objective but a means to serve economic progress. But here we are dealing with a medium-term, five-year plan, and in Hungary's given situation, described above in the introductory part of this article, it has proven useful to let the two foreign exchange balances to be alternatively maximized.

From the mathematical point of view, the construction of plan variants means in this case the alternative application of different objective functions and the alternative weighting of the various objectives by means of parametric programming. There is, of course, no obstacle from the methodological point of view to using also other objective functions in similar calculations and to represent thus other endeavours of economic policy as well when working out the plan variants.

2. The plan variants differ from each other in the extent to which the export targets were considered in the program to have been determined beforehand. In some calculations the export targets already set on the basis of the traditional methods were simply taken over and prescribed as lower bounds. It was only in respect of the additional exports over and above the lower bounds that the model was given freedom of choice. In some other calculations the lower bounds were prescribed only for one of the two market types. Finally, in a third group of the calculations it was assumed that we were free to re-plan the whole volume of exports, with no regard to already existing international obligations. This latter assumption is, of course, not realistic; the investigations may, nonetheless, be useful in providing a basis for the formulation of a Hungarian standpoint in the preliminary negotiations on future international agreements.

3. In the case of a part of the estimates — e.g. some export constraints — we were uncertain. These estimates were therefore given, instead of one, two

values, a more optimistic and a more pessimistic one, with the calculations carried out for both assumptions.

From the methodological point of view, the calculations of the types 2 and 3 meant the numerical modification of the right-side constraints, i.e. the repetition of the calculations with the constraint system

$$\mathbf{A}\mathbf{x} = \mathbf{b}_k \tag{3}$$

with a constant **A** coefficient matrix but with varying constraint vectors $\mathbf{b}_1$, $\mathbf{b}_2$, etc.

Utilization of the computation series

It must be frankly admitted that according to some opinions a computation series of this type would cause only trouble and uncertainty. What is the use for the economic administration of this great number of simultaneous variants? Would it not be simpler if the electronic computer produced but one proposal whose acceptance the mathematical economists could unequivocally recommend? Yet, it is precisely here, in the possibility of choice, that lies the greatest importance of mathematical programming. This does, of course, not mean that without any comment and weighting, 43 plan variants should be submitted to the higher authorities as alternatives of equal rank. Preliminary selection of a kind will be required and a comparative evaluation from the economic point of view of the really significant alternatives. This working method, with its simultaneous consideration of various alternatives, may at first seem unusual; we are, however, convinced that it will be worth while to undertake the additional work it involves.

Let us now survey the additional advantages provided by the computation series carried out with the mathematical programming model, as against the methods of traditional planning.

1. It ensures, first of all, that *all plan variants should be realistic from the point of view of the basic proportions*, which in the terms of mathematical programming means that the programs should be feasible and satisfy the system of constraints. This requirement, evident as it may be, cannot be easily ensured by means of the traditional methods. As mentioned above, the traditional methods used in working out the third five-year plan did not lead to a fully consistent plan. The last official plan proposal, preceding the computations carried out on the basis of the model, was originally not a feasible program. There appeared unsold surpluses in some sectors and shortages in others.

The feasibility of a plan obtained by means of programming will, of course, be closely connected with the degree of aggregation in the model. The feasibility of a program obtained by means of a highly aggregate model will not exclude the possibility of potential equilibrium disturbances in the partial relationships neglected in the model. If for no other reason, also less aggregate models are needed. We shall revert to this question below.

2. *By means of our mathematical programming model, so-called efficient programs can also be constructed.* A program will be called efficient if it cannot be confronted with another program which is more advantageous from several points of view, only with another program which is more favourable from one point of view and less favourable from another. For example, program $\mathbf{x}_1$ may be efficient if there can be found another program $\mathbf{x}_2$ which is more favourable as regards the balance of socialist foreign exchange but less favourable as regards that of capitalist foreign exchange. But program $\mathbf{x}_3$ will not be considered efficient if either program $\mathbf{x}_1$ or program $\mathbf{x}_2$ are more favourable from the point of view both of the socialist and the capitalist balances of foreign exchange.*

To produce efficient programs is a more modest and more realistic requirement than to find the "optimum" program. In the above sense, all of the 43 plan variants produced by us were efficient programs.

3. *The computation series will help to find the comparatively stable points in the national economic plan, those which are relatively less sensitive to the various endeavours of economic policy and to alternative assumptions*, and also the non-stable points which are sensitive to modifications in the said factors. It is for this reason that the calculations of this type are called *sensitivity analysis.*

In our computations, the sensitivity of the programs has been analysed in several ways. The minimum and maximum values of the production and foreigntrade forecasts were worked out for the different variants and the stability of the forecasts was characterized by the interval between the two values. The analysis revealed e.g. that in coal mining or the building industry the production forecasts were comparatively stable; the difference between the maximum and the minimum values was 2.2 per cent for the former and 0.1 per cent for the latter industry. On the other hand, the forecasts of the engineering or the textile industry were comparatively unstable.

In another type of analysis, we calculated the standard deviation and the variation coefficients (standard deviation as a percentage of the mean) of the forecasts referring to identical activities in the various plan variants. It appeared that the forecast of the total production of the national economy was comparatively stable, with a variation coefficient of 0.4 per cent. The foreigntrade forecasts, on the other hand, were considerably more sensitive, the variation coefficient being 6.6 per cent for socialist imports, 12.3 per cent for capitalist imports, 7.4 per cent for socialist exports and 10.2 per cent for capitalist

* With the given constraints $\mathbf{A}\,\mathbf{x} = \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$, all programs belonging to the vertices of the set of feasible programs as well as their convex combinations will be considered efficient programs.

exports. Going more thoroughly into the question and analysing the forecasts in the 18-sector breakdown, we will find the deviations even more marked. For example, the exports of the food industry to capitalist markets represent a comparatively stable activity with a 5 per cent variation coefficient, whereas the capitalist exports of the chemical industry are highly sensitive to the choice of both the aims of economic policy and the foreign-trade forecasts, and the variation coefficient is here 166.3 per cent.

The sensitivity analysis of the programs helps the planners to separate the problems needing a more thorough investigation from those which can be relatively easily clarified. It will not be worth while to discuss at great length the comparatively stable forecasts; the mental energies of the planners can be concentrated on the investigation of sensitive points.

4. By means of the computation series it will be possible to demonstrate the consequences of the alternatives concerning the basic decision problems of economic policy. This point should be illustrated by an example.

Let the objective function be the combined optimization of consumption and the balance of capitalist foreign exchange. Let the lower bounds both on consumption and on the positive balance of capitalist foreign exchange be set at the level prescribed in the official plan proposals. The mathematical programming model will be able to reveal some surplus as against this level; the surplus will materialize either in additional private consumption or in a dollar surplus of the foreignexchange balance or in a combination of the two. It is thus possible to work out the "opportunity cost" of additional consumption, i.e. the amount of the balance in Dollar terms, to be renounced in order to maximize consumption. Or, conversely, the amount of additional consumption, in Forint terms, to be renounced when the whole surplus is used to improve the balance of capitalist foreign exchange. With the export targets of the official plan proposals prescribed as lower bounds, the calculations have shown this index to be 61 Forints per Dollar. This is the "price" of improving the Dollar balance as expressed in Forints of additional consumption lost.

The mathematical programming model does not take a stand on the question in what proportion should the surplus attainable over and above the official plan proposals be allocated to consumption, to the improvement of the balance of capitalist or socialist foreign exchange, or to some other purposes (e.g. investments etc.). But it reveals instead what will be the consequences if a decision were taken on the question by the economic administration; what is more, it reveals the consequences for the case when the decision is supported by the most efficient plan, by the comparatively most favourable program from the point of view of the basic decision. Accordingly, the top organs need not discuss such questions as e.g. the desirable production volume of the engineering or the food industry. These details will be worked out by means of the mathematical model. Instead, they will have to decide on the final and fundamental questions of economic policy. *The computation series will thus raise the decision problems of economic administration to a higher level, to that of the fundamental questions.*

On the shadow prices

Each of the calculated plan variants represents the primal program of a linear programming problem. Together with these we have, naturally, calculated also the dual programs, i.e. the system of shadow prices belonging to each individual plan variant.

Since the absolute magnitude of the shadow prices depends also on the economic contents and measurement unit of the objective function, for the purposes of comparison we have also computed the shadow price ratios, the relative shadow price systems. The shadow price of the final domestic demand in the food industry was taken throughout as numeraire and the relative shadow prices of all other constraints were computed.

As an important inference drawn from the computation it may be stated that the relative shadow price systems are unstable. We must not believe we are dealing here with some "objectively determined" valuations. *The relative shadow prices will to a great extent depend on the aims set by economic policy under the given economic conditions and on the assumptions concerning the uncertainties of the future.*

Let us give an example. We have examined the rate of substitution of fixed capital and labour in our model. The shadow price of the fixed capital quota expresses the increase in the value of the objective function brought about by a unit increase in the fixed capital quota. This will thus show the marginal efficiency of fixed capital. The shadow price of the manpower quota expresses the increase in the value of the objective function brought about by a unit increase in the manpower quota. This will, accordingly, give the marginal productivity of manpower, of live labour. The quotient of the two shadow prices, which we denoted R, expresses the marginal rate of substitution between manpower and fixed capital manpower:

$$R = \frac{\dfrac{\partial C}{\partial K}}{\dfrac{\partial C}{\partial L}} \tag{4}$$

where K is the fixed capital quota and L the manpower quota.

This index has a great economic importance. An analogous one can usually be derived from the various aggregate macroeconomic production functions. The view is widely held that it will be determined exclusively by technological factors: the technological possibilities of combining fixed capital with live labour. Should the shadow prices be used for the purpose of price calculations, it is this index that would have to determine the ratios between the rental on capital and the charges based on and proportionate to labour (wages and pay-roll taxes).

In the course of our serial computations the quotient R has been worked out for all programs obtained from one of the parametric programmings. In these calculations it was assumed that exports were not bound by previously planned export transactions. With parameter $\lambda = 0$, optimizing "purely" the balance of capitalist foreign exchange, the marginal rate of substitution, R was 4800 forints per head, which means that 4800 forints of additional fixed capital can substitute an additional unit of manpower. With parameter $\lambda = 1$, maximizing "purely" consumption, the marginal rate of substitution R was 12 400 forints per head. With intermediary values of the parameter the value of R will fall between the two figures.

It has thus turned out that *the marginal rate of substitution depends not only on technological factors but also on those of economic policy*, on the objectives for the attainment of which we wish to mobilize the reserves of the national economy. If the surplus obtained over and above securing a minimum level of consumption and of the balance of foreign exchange is used for increased consumption, then labour will become comparatively more scarce. If, on the other hand, we concentrate on improving the foreign-trading position, then fixed capital will become a relatively narrower bottleneck. Nor is the difference insignificant; the rate of substitution is highly sensitive to this choice. In the case of maximizing consumption its value is more than double that obtained in the case the balance of foreign exchange is maximized. In terms of price formation this means that (provided that wages and pay-roll taxes are identical in the case of both economic policies), the plan optimizing the balance of foreign exchange will have to be supported by a rental on capital more than double that required in the case the plan maximizes consumption.

The experiences in connection with the sensitivity of the system of shadow prices are most thought-provoking. The proposals suggesting the direct use of shadow prices in price formation are generally known. Some would imagine this was a simple way. They think that because it is easy to find the only optimum plan, it will be simple to calculate the corresponding and equally unique optimum system of shadow prices. From what has been said above, it will be clear that this is by far not so simple. Precisely because there exists no evident, given and solely possible optimum primal plan, there can be no evident and given dual shadow price system either. The system of shadow prices is highly sensitive to the choice of economico-political aims. It is, therefore, certain that it will be possible to use the shadow prices obtained by programming in the calculation of actual prices only if (and this is but one of several qualifications) the economic administration has a clearly defined economic policy which may be expected to remain fairly stable over a longer period, and if this economic policy finds an adequate expression in the structure and the numerical data of the mathematical model.

The connection between aggregate and "two-level" programming

Parallelly with the calculations of the aggregate programming model, a numerous research team was engaged in work on the so-called "two-level planning" model of the 1966/70 plan.* In the two cases we were dealing with models of a related type, both from the economic and the mathematical points of view. In both bases the program determined the targets of production and foreign trade for 1970, the terminal year of the plan period, and, indirectly, the pattern of investment activities for the period 1966 to 1970. The economic contents of the objective functions and of the constraints in the two models were also related. There are, however, also essential differences between the two models.

In the aggregate model production is broken down into 18 sectors, i.e. 18 large product aggregates. The two-level model, on the other hand, has 505 much less aggregate product groups. Accordingly, the aggregate model has some 80 and the two-level model some 2500 variables; the former contains some 80 and the latter some 2000 equations. One important difference is thus in the degree of aggregation.

In the aggregate model there are no technological alternatives or technical variants, whereas the two-level model enables the choice also between these types of variants. The latter model thus lends itself not only for the planning of sectoral proportions and the product pattern but also for that of technical development.

The aggregate model — similarly to the input-output table on which it is based — embraces, if only in an aggregate form, the whole domain of social production. The two-level program, though much more detailed, is more restricted in scope, planning as it does only the production of 505 priority products allocated on the basis of the balances of the National Planning Office, and the related investment and foreign trading activities.

In the aggregate model — as in the input-output table on which it is based — social production is accounted for in value terms, at current prices. In the construction of the two-level model, however, it was endeavoured to eliminate the measurements at current prices, in order to avoid the well-known distortions of the price system in force. Wherever physical units could be used

* The research team engaged in two-level planning is headed by J. Kornai, one of the co-authors of this article. For details on the research work see [3], [4] and [5]. The methodology and the numerical results of the national economic programming project were described in hectographed publications, of which 22 volumes have appeared so far.

and are also used by the National Planning Office, the model employed such units. Current prices were used only for the product groups which could not be measured in physical units.

These comparisons will make it clear that we are dealing here not with two competing models but with research projects which complement each other. It will not be worth while to put the question, which of the two model types is "better", because each of them has its own advantages which must be paid for by certain disadvantages. The advantage of a high degree of aggregation lies in easier manageability; it is thus simpler to work out a great number of plan variants and to carry out a great variety of sensitivity analyses. The large model is more cumbersome, but will be less distorted by the simplifications due to aggregation. With the other differences, the case is similar. Completeness is most important, but an accentuation of the major product flows and the most important investment activities has also its justification. It is one of the rational traits of traditional planning that it aims simultaneously at full-range planning and the detailed planning of priority products and the major investment activities. As regards the units of measurement, the avoidance of current prices and the elimination of their distorting effects has much to recommend it. However, by the exclusive use of physical measurements, a simple summarization of the various targets becomes impossible.

Owing to its manageability, the aggregate model has increased importance in the early stages of planning, it the initial calculations. In that stage it is of utmost importance to work out a great number of variants and to analyse their characteristics and consequences. With the progress of the planning work we will have to choose from among the great number of variants and work out the details for the few basic ones only. In this later phase, the disaggregate two-level model will come to the fore, which enables us to obtain a plan feasible also in its details and taking into account the special potentialities of the individual sectors.*

On the basis of what has been said, we believe that both model types have a place in working out the foundations of the five-year plans. (Not to mention the fact that there is ample justification for the use of further model types, such as the even more highly aggregate non-linear growth models with few variables, the multi-period programming models, and others.)

The analysis of the similarities and divergencies between the aggregate and the detailed, two-level linear programming models suggests the idea to

* Utilization of the aggregate model in the early stages of planning will be the more possible as its construction does — contrary to the practice followed up to now — not indispensably require a plan proposal worked out on the basis of traditional methods, nor an input-output table derived from the latter. In the course of the present research work we have already attempted to give a forecast of the 1970 planned table on the basis of another source, the statistical table for 1961, at least for the sake of comparison. It will be worth while to continue the research work in this direction.

make an attempt at the combination of the two model types. From the mathematical point of view this should be entirely possible, as both contain linear equations. From the economic point of view, the linking of the two models would mean that within the framework of a large equation system one part of the equations would ensure the assertion of the main aggregate proportions and the other part of the equations a harmonious development of the detailed disaggregate relationships. The equations describing the aggregate and disaggregate relationships could be linked together by means of various aggregate and disaggregate variables and equations.

The real difficulties of the combination may be expected to arise in connection with the quantification of these aggregate and disaggregate variables and equations. We will have to attempt, e.g., to describe in the form of linear relationships the connection between a sector's total production value and the production volume of its major products as measured in physical units. This difficulty, now brought to the fore by the problem of linking together the two model types, was actually concealed already in the practice followed by traditional, non-mathematical planning up to now. This practice has, at least in Hungary, never really penetrated into the question, whether the necessary proportions are ensured between the aggregate production indices (expressed usually in value terms) and the more detailed indices (measured frequently in physical units). These interrelations need further detailed investigation.

In conclusion, one more remark concerning the combination of the two models. From our endeavours to link the aggregate model based on the input-output table and the detailed two-level model we expect, among other things, also to bring about an approach between the representatives of the two "schools" of mathematical planning, who have up to now gone somewhat separate ways, the "input-output economists" and the "programming economists". The linking of the two model types will provide a basis for closer cooperation between the two groups in improving the methods of planning.

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ПРИМЕНЕНИЕ СВОДНОЙ МОДЕЛИ ПРОГРАММИРОВАНИЯ В ПЛАНИРОВАНИИ ПЯТИЛЕТОК

Я. КОРНАИ — Л. УЙЛАКИ

В статье представляется очень укрупненная 18-секторная модель программирования, которая была использована при исчислении наметок пятилетнего плана в области капитальных вложений, производства и внешней торговли. При составлении модели исходили из таблицы «затраты — выпуск», в которой отображались потоки продукции между секторами. Эта таблица была расширена дальнейшими столбцами и строками и преобразована в модель математического программирования.

Переменные модели представляют собой производственную и внешнеторговую деятельность последнего года плана. Системой условий помимо баланса «затраты — выпуск» по продуктам ограничивается использование капитальных ресурсов, живого труда и природных ресурсов, а также — на основании рыночных соображений — определяется верхний предел некоторым внешнеторговым деятельностям. Было применено несколько альтернативных целевых функций, как максимизация потребления населения и оптимизация сальдо внешнеторгового баланса. Эти цели применялись и комбинированно — в рамках параметрических программирований.

В статье оспаривается точка зрения, согласно которой целью математического программирования является определение единственной «оптимальной» программы.

Вместо этого следует производить расчет большого числа плановых вариантов и представить их для принятия политического решения.

Затем в статье анализируются полученные в рамках серии расчетов теневые цены, и констатируется, что система теневых цен оказалось весьма чувствительной к принятой в модели за основу экономической политике, а также к предположениям о неопределенных данных.

Наконец, авторами излагается взаимосвязь сводного программирования и детализированного «планирования на двух уровнях», их взаимоотношение.