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COMPUTING LINEAR PROGRAMING PROBLEMS WITH AN APPROXIMATION METHOD BASED ON DECOMPOSITION

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1. Introduction

For the solution of large-size linear programing problems, it may be useful to resort to what is called decomposition algorithms. A number of methods have been worked out in recent years. Experiences have, however, shown that slow convergence to the optimum is a common characteristic of all these methods.[1)]

In the following, an approximation method will be presented.[2)]

The underlying mathematical concept is not original; the procedure may be considered as a naive heuristic variant of the Dantzig-Wolfe [2] decomposition algorithm - in the following: the DW-method). It cannot guarantee that the optimum of the original problem before decomposition will be reached; it may, however, help to obtain , in the first iteration already, programs with a comparatively favourable objective function value, lending themselves to practical interpretation.

The subject will be treated as follows :

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1) No investigations are known to have been carried out so far with the aim of comparing on the basis of genuinely representative computation series (i.e. problems of sufficient size and variety of structure) the practical computing-thechnical efficiency of the various non-decomposition and decomposition, exact and approximative methods of linear programing.

2) For a first description see [4] .

In Chapter 2, definitions will be given and assumptions presented. Chapter 3 will describe the approximation method in general form. Chapter 4 recommends some computational "triks" to increase the efficiency of the procedure. In Chapter 5, statements are made on the characteristics of the method. Finally, in Chapter 6, one of the possible economico-sociological interpretations of the method will be presented.

2) Definitions and assumptions

2.1. Two-level structure

We have a linear programing problem. It is possible without limiting generality to deal only with the case where the system of contraints consists exclusively of inequalities. The problem will be of a two-level structure when the variables can be arranged in the form presented in formula (1).[3]

[3] Notations. Bold capital letters denote matrices, bold small letters vectors, cursive capital letters sets, and cursive small letters real numbers. The prime beside the symbols is the sign of transposition. The prime beside the symbol of a vector denotes the row vector. The star is the sign of optimality. Symbol $\underline{\underline{E}}$ denotes the unit matrix, $\underline{\underline{1}}$ a summarizing vector, always in the dimension corresponding to the formula in question. Empty sets are denoted $\emptyset$. As we are dealing here exclusively with linear relationships, the number on the place of the exponenet is in each case an upper index.

$$
\begin{array}{llr}
 & \underline{\underline{A}}_1\underline{\underline{x}}_1 + \underline{\underline{A}}_2\underline{\underline{x}}_2 + \ldots + \underline{\underline{A}}_n\underline{\underline{x}}_n \leq \underline{\underline{b}}_o & \text{(1a)} \\
 & \underline{\underline{B}}_1\underline{\underline{x}}_1 \qquad\qquad\qquad\qquad \leq \underline{\underline{b}}_1 & \text{(1b)} \\
\text{(1)} & \qquad \underline{\underline{B}}_2\underline{\underline{x}}_2 \qquad\qquad\qquad \leq \underline{\underline{b}}_2 & \\
 & \qquad\qquad \ddots \qquad\qquad\qquad \vdots & \\
 & \qquad\qquad\qquad\quad \underline{\underline{B}}_n\underline{\underline{x}}_n \leq \underline{\underline{b}}_n & \\
 & \underline{\underline{x}}_1 \geq \underline{\underline{o}},\ \underline{\underline{x}}_2 \geq \underline{\underline{o}}, \ldots\ \underline{\underline{x}}_n \geq \underline{\underline{o}} & \text{(1c)} \\
 & \underline{\underline{c}}'_1\underline{\underline{x}}_1 + \underline{\underline{c}}'_2\underline{\underline{x}}_2 + \ldots + \underline{\underline{c}}'_n\underline{\underline{x}}_n \rightarrow \max! & \text{(1d)}
\end{array}
$$

In the following, problem (1) will be called the large--zize problem.

To work out the two-level structure , the activity variables of the large-size problem have been arranged to form n units wich will be called sectors.[4]

Let $\underline{\underline{x}} = [\underline{\underline{x}}'_1, \underline{\underline{x}}'_2, \ldots, \underline{\underline{x}}'_n]$ denote the program vector of the large-size problem and x^* the optimum program.

The constraints can be divided into two groups. Group (1a) comprises the central constraints where non-zero coefficients may

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[4] The present article adheres, as far as possible, to the terminology used in [3] , dealing with two-level planning, and introduced in the first experimental economy-wide programing project connected with the drawing-up of the 1966-70 plan. For details of the latter see [6] .

be found among the variables of at least two sectors. Let us denote the number of central constraints m . Group (1b) comprises the special sector constraints where non-zero coefficients may be found exclusively in the sector concerned.

Let X denote the set of feasible programs of the large size problem.

First assumption. Set X is bounded and non-empty; $X \neq \emptyset$.

Let us call matrix $\underline{\underline{U}}$ central constraint allocation:

(2) $\underline{\underline{U}} = [\underline{\underline{u}}_1, \underline{\underline{u}}_2, \ldots, \underline{\underline{u}}_n]$.

where $\underline{\underline{u}}_i$ $(i=1,\ldots,n)$ is equal in size with vector $\underline{\underline{b}}_o$, which means that the number of its components is m .

Let us call the i-th sector problem belonging to $\underline{\underline{U}}$ central constraint allocation, a linear programing problem with variable x_i and constraints (3)-(4)-(5) - but at least (4)-(5) - below :

(3) $\underline{\underline{A}}_1 \underline{\underline{x}}_1 \leqq \underline{\underline{u}}_i$,

(4) $\underline{\underline{B}}_i \underline{\underline{x}}_i \leqq \underline{\underline{b}}_i$,

(5) $\underline{\underline{x}}_i \geqq \underline{\underline{o}}$:

2.2. The degrees of feasibility and optimality.

Let us call $[\underline{\underline{u}}_i, \underline{\underline{b}}_i]$-feasible the sector program $\underline{\underline{x}}_i$ which satisfies the conditions (3)-(4)-(5). Let $X_i[\underline{\underline{u}}_i, \underline{\underline{b}}_i]$ denote the set of these programs.

Let us call evaluable central constraint allocation the matrices $\underline{\underline{U}}$ which satisfy the following two conditions:

(6) $$\underline{\underline{u}}_1 + \underline{\underline{u}}_2 + \ldots + \underline{\underline{u}}_n \leqq \underline{\underline{b}}_c \ ,$$

(7) $$x_i\left[\underline{\underline{u}}_i, \underline{\underline{b}}_i\right] \neq \emptyset \quad \text{for any} \quad i \quad (i=1,\ldots,n)$$

Let U denote the set of evaluable central constraint allocations.[5] From the first assumption, it will be obvious that

(8) $$U \neq \emptyset \ .$$

Sector program x_i will be $\left[\underline{\underline{u}}_i, \underline{\underline{b}}_i, \underline{\underline{g}}_i\right]$ - optimal if it constituted the optimum solution of the following sector problem:

(9) $$\begin{aligned} \underline{\underline{A}}_i \underline{\underline{x}}_i &\leqq \underline{\underline{u}}_i \\ \underline{\underline{B}}_i \underline{\underline{x}}_i &\leqq \underline{\underline{b}}_i \\ \underline{\underline{x}}_i &\geqq \underline{\underline{o}}_i \\ \underline{\underline{g}}_i' \underline{\underline{x}}_i &\rightarrow \max ! \end{aligned}$$

where $\underline{\underline{g}}_i$ may be any objective-function coefficient vector (even different from $\underline{\underline{c}}_i$ figuring in the large-size problem).

---

5) For the propositions and statements relating to the decomposition of the large-size problem into a two-level one, and to evaluable central constraint allocation see [3], p. 144-150.

Sector program $\underline{\underline{x}}_i$ will be $[\underline{\underline{b}}_i]$ -feasible if it satisfies the following constraint system :

$$
\begin{aligned}
&\underline{\underline{B}}_i \, \underline{\underline{x}}_i \leqq \underline{\underline{b}}_i \\
(10) \qquad &\underline{\underline{x}}_i \geqq \underline{\underline{c}} \, .
\end{aligned}
$$

Let $\underline{\underline{X}}_i [\underline{\underline{b}}_i]$ denote the set of $[\underline{\underline{b}}_i]$ -feasible programs.

Sector program $\underline{\underline{x}}_i$ will be $[\underline{\underline{b}}_i, \underline{\underline{g}}_i]$ -optimal when it constitutes the optimal solution of the following problem :

$$
\begin{aligned}
&\underline{\underline{B}}_i \, \underline{\underline{x}}_i \leqq \underline{\underline{b}}_i \\
(11) \qquad &\underline{\underline{x}}_i \geqq \underline{\underline{o}} \\
&\underline{\underline{g}}_i' \, \underline{\underline{x}}_i \rightarrow \max !
\end{aligned}
$$

2.3. The comparative program

In order to employ the approximation method efficiently, it will be useful to know one program of the large-size problem. This will be called the comparative program and denoted $\underline{\underline{x}}^o = [\underline{\underline{x}}_1^{o'}, \underline{\underline{x}}_2^{o'}, \ldots, \underline{\underline{x}}_n^{o'}]$. The name indicates the fact that it is with this program that the programs obtained in the course of computation will be compared. [6)]

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6) In the first experimental Hungarian economy-wide programing project, the so-called official program - worked out by practical planners on the basis of non-mathematical, traditional planning methods, independently of our model - was used as the comparative program.

Second assumption. $\underline{\underline{x}}^o \in X$.

Let us call comparative central constraint allocation the matrix $\underline{\underline{U}}^0$ whose i-th column vector is determined in the following way :

$$u_i^o = \underline{\underline{A}}_i \, \underline{\underline{x}}_i^o \qquad i = 1, \ldots, n . \tag{12}$$

From the first and second assumption as well as from proposition (8) it follows that the comparative central constraint allocations are evaluable, i.e.

$$\underline{\underline{U}}^o \in U . \tag{13}$$

Third assumption . $\underline{\underline{c}}' \underline{\underline{x}}^o < \underline{\underline{c}}' \underline{\underline{x}}^*$

Fourth assumption. $\underline{\underline{c}}' \underline{\underline{x}}_i^o < \underline{\underline{c}}_i' \underline{\underline{x}}_i^1$ for any i[7] where $\underline{\underline{x}}_i^1$ constitutes the optimum program of the following problem :

---

[7] On the basis of our practical experiences it is justify to declare the fourth assumption valid for any i . Up to the present, we have not met with any comparative program which would have constituted the optimum solution of problem (14) .

$$(14)\qquad \begin{cases} A_i x_i \leqq u_i^0 \\ B_i x_i \leqq b_i \\ x_i \geqq 0 \\ c_i' x_i \longrightarrow \max ! \end{cases}$$

Vector x_i^1 will be called the sector-optimal program; this this is the optimum program computed with the original vector of objective-function coefficients in the case of comparative central constraint allocation.

2.4. The plan proposal

Let us denote t_i and call the plan proposal of the i-th sector the following vector :

$$(15)\qquad t_i = A_i x_i \qquad\qquad x_i \in X_i \left[b_i \right] .$$

Let us denote s_i and call the objective function contribution of the i-th sector following real number :

$$(16)\qquad s_i = c_i' x_i \qquad\qquad x_i \in X_i \left[b_i \right] .$$

In the following, the plan proposals will be given serial numbers in each sector and the same serial number be given to the objective-function contributions belonging to them The upper index beside the symbol indicates these serial numbers.

Two special plan proposals will be described together with their objective-function contributions: the comparative and the sector-optimal plan proposals.

$$(17)\qquad t_i^o = A_i x_i^o = u_i^o \qquad\qquad s_i^o = c_i' x_i^o$$

$$i=1, \ldots, n$$

$$(18)\qquad t_i^1 = A_i x_i^1 \qquad\qquad s_i^1 = c_i' x_i^1 .$$

3. General description of the procedure

In the description, double numbering will be employed: the first number is that of the iteration, the second one that of the step within the iteration.

In accordance with the usual interpretation of decomposition methods, it will be assumed that part of the steps is carried out by the centre, and the other part by the sectors. In a definite number of steps, information flows from the centre to the sectors or conversely.[8] Accordingly, in the case of each step it will be indicated whether it has to be carried out in the centre or in

---

[8] If our process were investigated only from the computing -technical point of view, all operations could, of course, be carried out by the same group of planners with the same computer. In that case, the terms "centre" and "sector" would refer only to different phases in the organization of the work.

sectors. Should a transmitting of information take place in the step in question, its direction will be indicated.

Some steps of the process require the solution of an exactly formulated mathematical problem. These operations will be carried out in practice on the computer. Other steps, on the other hand, will have to be carried out by the practical planners without any exact algorithm, in a heuristic-intuitive manner. It will, accordingly, be indicated with every step whether the problem in question is an algorithmic (A) or an heuristic (H) one. In the case of the heuristic steps, their formal contents will now be only described. Later on, we will revert to the information the planners may rely on when carrying out these steps.

It is being assumed that comparative program $\underline{\underline{x}}_i^o$ is known in every sector.

3.1. The first iteration

Step 1.1. (In the sectors; algorithmic). The value of $\underline{\underline{u}}_i^o$ is determined according to formula (12). In the knowledge of this, problem (14) is solved and sector-optimal program $\underline{\underline{x}}_i^1$ determined.

On the basis of the comparative and the sector-optimal program, plan proposals $\underline{\underline{t}}_i^o$ and $\underline{\underline{t}}_i^1$ are determined according to formulae (17) and (18), together with objective function contributions z_i^o and z_i^1 belonging to them.

Step 1.2. (In the sectors; heuristic.) The determination of vector pairs $\left[\underline{\underline{u}}_i^k, \underline{\underline{g}}_i^k\right]$) $k=2, 3, \ldots, K_i$ $\{1\}$.

Step 1.3 (In the sectors; algorithmic.)

The solution of the following sector problems on the basis of the constraints and objective functions determined in Step 2.

$$
\begin{aligned}
\underline{\underline{A}}_i \underline{\underline{x}}_i &\leqq \underline{\underline{u}}_i^k \\
\underline{\underline{B}}_i \underline{\underline{x}}_i &\leqq \underline{\underline{b}}_i \\
\underline{\underline{x}}_i &\geqq \underline{\underline{c}} \\
\underline{\underline{g}}_i^{k,} \underline{\underline{x}}_i &\longrightarrow \max!
\end{aligned}
\qquad k=2, 3, \dots, K_i \{1\} \tag{19}
$$

Let $\underline{\underline{x}}_i^k$ denote the optimum program of problem (19). On the basis of this, let us generate plan proposal $\underline{\underline{t}}_i^k$ as well as the objective function contribution c_i^k belonging to it.

Step 1.4. (In the sectors; algorithmic.)

On the basis of the results obtained in Steps 1 and 3, let us formulate the following matrix of plan-proposals and vector of objective function contributions :

$$\underline{\underline{T}}_i \{1\} = \left[\underline{\underline{t}}_i^o, \underline{\underline{t}}_i^1, \underline{\underline{t}}_i^2, \dots, \underline{\underline{t}}_i^{K_i \{1\}}\right]. \tag{20}$$

$$\underline{\underline{s}}_i' \{1\} = \left[s^o, s^1, s^2, \dots, s^{K_i \{1\}}\right]. \tag{21}$$

Step 1.5. (From the sectors to the centre).

The transmitting of matrices $\underline{\underline{T}}_i \{1\}$ and vectors $\underline{\underline{s}}_1' \{1\}$.

Step 1.6. The following central problem must be solved [9)]

Constraints (22b) will be called combination constraints.

$$\underline{\underline{T}}_1\{1\}\underline{\underline{y}}_1\{1\} + \underline{\underline{T}}_2\{1\}\underline{\underline{y}}_2\{1\} + \ldots + \underline{\underline{T}}_n\{1\}\underline{\underline{y}}_n\{1\} \leqq \underline{\underline{b}}_o \qquad (22a)$$

$$(22)\quad \begin{array}{ll} \underline{\underline{1}}'\underline{\underline{y}}_1\{1\} = 1 & \\ \underline{\underline{1}}'\underline{\underline{y}}_2\{1\} = 1 & (22b) \\ \quad\vdots & \\ \underline{\underline{1}}'\underline{\underline{y}}_n\{1\} = 1 & \end{array}$$

$$\underline{\underline{y}}_1\{1\} \geqq \underline{\underline{c}},\ \underline{\underline{y}}_2\{1\} \geqq \mathbf{o},\ \ldots,\ \underline{\underline{y}}_n\{1\} \geqq \mathbf{o} \qquad (22c)$$

$$\underline{\underline{s}}_1'\{1\}\underline{\underline{y}}_1\{1\} + \underline{\underline{s}}_2'\{1\}\underline{\underline{y}}_2\{1\} + \ldots + \underline{\underline{s}}_n'\{1\}\underline{\underline{y}}_n\{1\} \rightarrow \max! \qquad (22d)$$

The role of constraints (22 a) is analogous with that of constraint (1a) in the large-size problem; accordingly, we will call them here, too, central constraints.

Weights $\underline{\underline{y}}_i\{1\}$ are the variables of the central problem, vectors composed each of $(1 + K_i\{1\})$ conmponents:
$\underline{\underline{y}}\{1\} = [\underline{\underline{y}}_1'\{1\}, \underline{\underline{y}}_2'\{1\}, \ldots, \underline{\underline{y}}_n'\{1\}]$.

Let $\underline{\underline{y}}\{1\}^*$ denote the optimum weight vector, the solution

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[9)] The structure of the central problem corresponds to the structure of the "extremal problem" in the DW-method. See formulae (5)-(8) in [2].

of the central problem in the first iteration.

Step 1.7. (In the centre; algorithmic).

Let us compute $D\{1\}$, the additional yield obtained in the first iteration :

$$(23) \qquad D\{1\} = \sum_{i=1}^{n} \underline{\underline{s}}'_1\{1\}\underline{\underline{y}}_i\{1\}^x - \underline{\underline{c}}'\underline{\underline{x}}^o$$

3.2. The further iterations

Let us now turn to the 2., 3., ..., (z-1) - th, z-th iteration. In this section, the z-th iteration will be described in general form.

The last, Z-th iteration will be described in Section 3.3.

Step z.1. (In the centre; algorithmic). We establish the degree to which the upper bounds have been exhausted in the central problem solved in the (z-1)-th iteration, and determine constraint utilization vector $\underline{\underline{r}}\{z\}$, the h-th component of which will be :

$$(24) \qquad r_h\{z\} = \frac{b_h - w_h\{z-1\}}{b_h} \qquad h = 1, \ldots, m$$

where $w_h\{z-1\}$ is the value of the residual variable figuring in the h-th constraint in the optimum solution of the central problem of the z-1 -th iteration.

When $r_h\{z\} = 1$, the constraint is tight. When $r_h\{z\} < 1$, the constraint is loose, and $r_h\{z\}$ indicates the degree of looseness.

Step z.2. (In the centre; heuristic). The qualitative evaluation of the components of vector $r\{\underline{\underline{z}}\}$ whose value is 1 ; the qualitative characterization of the degree of tightness. ("Very tight" , "somewhat tight", etc.)

Step z.3. (From the centre to the sectors) .

The transmitting of the central information obtained in Steps z.1. and z.2 , i.e. vector $\underline{\underline{r}}\{z\}$ and the qualitative evaluations of the degree of tightness.

Step z.4. (In the sectors; heuristic) .

Determining, on the basis of central information received in Step z.3. and of an analysis of sector programing carried out in earlier iterations, the new $[\underline{\underline{u}}_i^k , \underline{\underline{g}}_i^k]$ vector pairs $(k = K_i\{z-1\} + 1 , K_i\{z-1\} + 2 , \ldots , K_i\{z\})$.

The vector pairs will be determined according to the following four viewpoints of formulating the plan proposals:

A) Should the h-th constraint be loose in the central problem, but tight in the sectors according to experiences gained in previous iterations, then the corresponding u_h constraint may be increased as against the value prescribed in the earlier iterations. When determining the extent of the increase, the central constraint's degree of looseness may be taken into account.

B) Should the h-th constraint be tight in the central problem , but not very tight according to experiences gained in previous iterations, then the corresponding u_h constraint may be decreased as against the value prescribed in the earlier iterations. When determining the extent of the decrease, the central constraint's degree of tightness may be taken into account.

C) Should the h-th constraint have proven very tight in the sector according to experiences gained in previous iterations, then the constraint may be increased as against the value prescribed in the earlier iterations.
This can be done even in the case when the same constraint appeared tight in the central problem.

D) The minimization of the input of some tight constraint may be given as the objective function. This may also be the minimization of the joint input of several tight constraints with the averaging with suitably chosen weights the various inputs.

As a possible system of weights we may use the shadow prices belonging to the selected tight constraints in the (z-1) -th iteration, the optimum dual solution of the central problem.[10)]

To determine the new $[\underline{\underline{u}}_i^k, \underline{\underline{g}}_i^k]$ vector pairs whose total number will be $(K_i\{z\} - K_i\{s-1\})$ the viewpoints A-D] listed above can be combined in various ways[11)].

[10)] Accounting for all central constraint inputs at shadow prices and deducting this from the original $\underline{\underline{c}}_i$ vector, we will reach the objective function of the exact DW - method. This question will be dealt with in section 4.2.

[11)] In this article - for the sake of simplicity - we are dealing exclusively with the case where there are only upper bounds both in the large-size problem and in all sector problems. In actual practice this is not ./.

Spep z.5. (In the sectors; algorithmic) .
Sector problem (19) in solved on the basis of the constraints and objective functions determined in Step 4. On the basis of the optimum programs obtained, we generate the new $\underline{t}_i^k$ plan proposals and s_i^k objective function dintributions $(k = K_1\{z-1\} + 1, \ldots, K_i\{z\})$.

Step z.6. (From the sectors to the centre) . Transmitting the new plan proposals and objective function contributions.

Step z.7. (In the centre; algorithmic) .
The enlarged central problem is constructed. By the end of Step z.6, a total of $(1 + K_i\{z\})$ plan proposals concerning the i-th sector will have come in . Accordingly, in the enlarged problem, weight vector $\underline{y}_i\{z\}$ and objective function contribution vector $\underline{s}'_i\{z\}$ will contain $(1 + K_i\{z\})$ eomponents, and the plan proposal matrix $\underline{\underline{T}}_i\{z\}$ will have the same number of columns.

The enlarged central problem is solved; the optimum program will be $\underline{y}\{z\}$*.

./.

always the case. In the case of lower bounds, Step z.4. must be modified accordingly. Should there be e.g. a product balance among the central contraints, then a lower bound must be prescribed for the producing sector. In such cases, viewpoint B(of formulating the plan proposals should be applied with the modification that the lower bound is raised (whereas in the user sectors the upper bound in decreased in accordance with viewpoint B) .

Step z.8 (in the centre; algorithmic).

We are computing - in a manner analogous with formula (23) - $D\{z\}$, the objective function surplus achieved as against the comparative program.

Step z.9 (in the centre; heuristic).

Considering the value of $D\{z\}$. Should it prove unacceptable, the procedure is continued and the z+1 -th iteration carried out.

If it is acceptable, no further iteration will be carried out, and we will proceed to the concluding steps.

3.3. Concluding the procedure

Let the iteration be numbered Z where in the 9th step the decision is made not to carry out any further iteration. In that case, two concluding steps must still be made.

Step z.10 (From the centre to the sectors).

Transmitting the optimum program $\underline{\underline{y}}_i\{Z\}^*$ obtained in the 7th step of the Z-th iteration.

Step z. 11. (In the sectors; algorithmic).

We determine the improved sector program $\underline{\underline{x}}_i\{Z\}$:

$$\underline{\underline{x}}_i\{Z\} = \underline{\underline{X}}_i\{Z\}\,\underline{\underline{y}}_i\{Z\}^* , \tag{25}$$

where $\underline{\underline{X}}_i\{Z\}$ is a matrix whose column vectors are all $(1 + K\{Z\})$ sector programs $\underline{\underline{x}}_i^k$ computed up to now and serving as a basis for the plan proposals.

The ensemble of improved sector programs forms the improved program $\underline{\underline{x}}\{Z\}$:

(26) $\underline{\underline{x}}\ \{Z\} = [\underline{\underline{x}}_1'\{Z\},\ \underline{\underline{x}}_2'\{Z\},\ \ldots,\ \underline{\underline{x}}_n'\{Z\}]'$

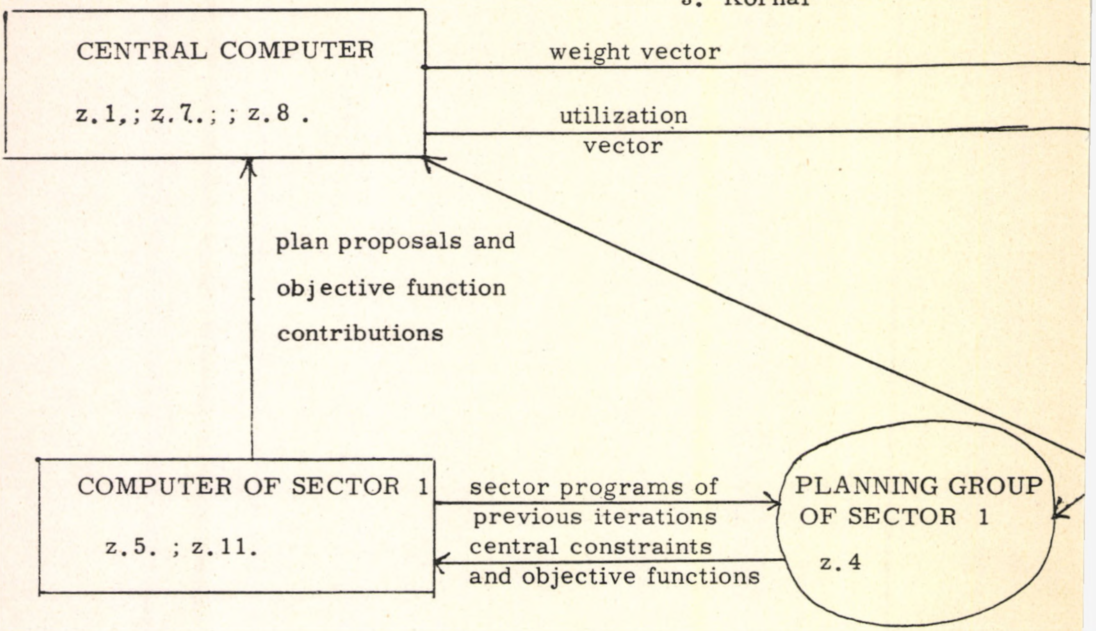
3.4. The scheme of information flows

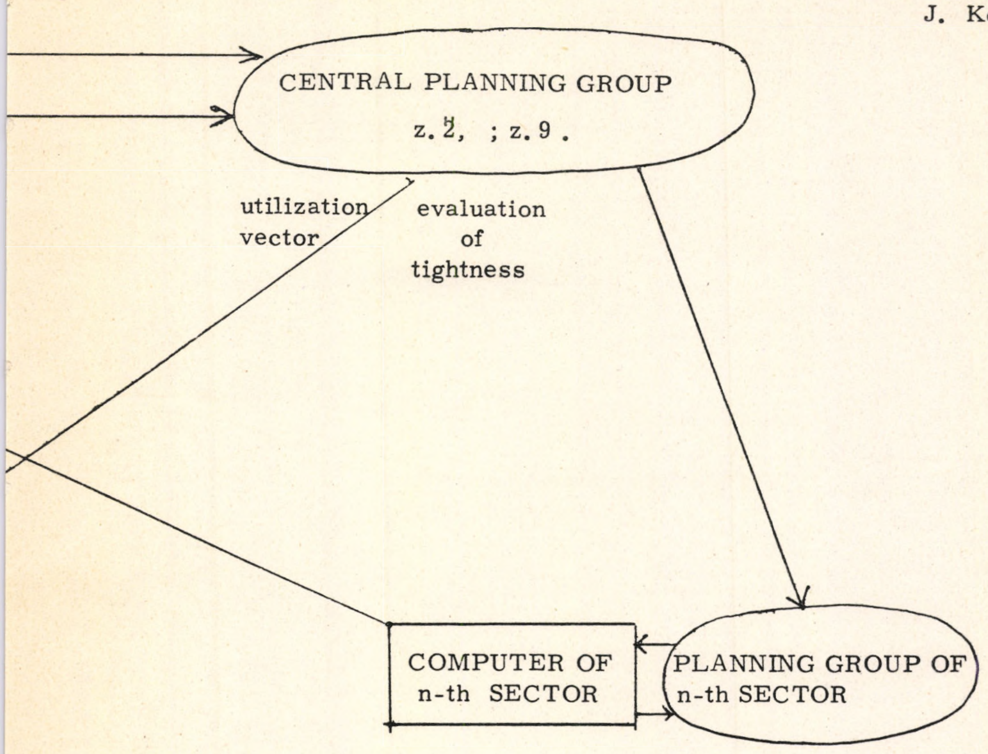
The procedure described in sections 3.1 - 3.3 is presented schematically. As in the description above, in this scheme , too; the usual concept of the interpretation of decomposition methods will be used, namely that part of the operations is carried out by the "centre" and the other part by the " sectors" .

Moreover, a further step will be made in the application, of the institutional interpretation. A distinction will be made between the living planners in the centre and the sectors, the mon employing the models and methods, on the one hand, and the " dead" machines with the data, instructions and algorithms fed into them , on the other .

In the scheme , circles represent the groups of planners in the centre and the sectors, i.e. the living men, and rectangles stand for the computers. The "circles" carry out the heuristic operations, the "rectangles" the algorithmic ones.

In the circles and rectangles we show the steps of the z-th iteration that are taking place there . To these, the contents of the information are also given.


CENTRAL COMPUTER
z. 1, ; z. 7. ; ; z. 8 .
weight vector
utilization vector
plan proposals and objective function contributions
COMPUTER OF SECTOR 1
z. 5. ; z. 11.
sector programs of previous iterations
central constraints and objective functions
PLANNING GROUP OF SECTOR 1
z. 4


CENTRAL PLANNING GROUP
z. 2, ; z. 9 .
utilization vector
evaluation of tightness
COMPUTER OF n-th SECTOR
PLANNING GROUP OF n-th SECTOR

4 Possibilities of modification

4.1. Some computational "tricks"

In the following, some possible modifications of the general methos describel in Chapter 3 will be pointed out which may increa= se the practical efficiency of the procedure.

1) In the DW-method, the central problem combines exclu= sively plan proposals of which no $[\underline{\underline{u}}_i, \underline{\underline{b}}_i, \underline{\underline{g}}_i]$ -optimality but only $[\underline{\underline{b}}_i, \underline{\underline{g}}_i]$ -optimality is required.

This is fessible also with the approximation method, provi= ded that there exist already one or two ensembles of plan proposals which are $[\underline{\underline{u}}_i, \underline{\underline{b}}_i, \underline{\underline{g}}_i]$ -optimal with some $\underline{\underline{U}} \in U$.

2) It is not absolutely necessary to use exclusively $[\underline{\underline{u}}_i, \underline{\underline{b}}_i, \underline{\underline{g}}_i]$ -optimal or $[\underline{\underline{b}}_i, \underline{\underline{g}}_i]$ -optimal programs when formu= lating the plan proposals.

These can be supplemented in every sector by some $[\underline{\underline{u}}_i, \underline{\underline{b}}_i]$ -feasible and $[\underline{\underline{b}}_i]$ -feasible (but not optimal) programs.

They may be considerably easier generated than the optimum sector propgrams.

3) The approximation of the optimum solution of the large-si= ze problem may become more difficult when the central problem can combine only $\underline{\underline{t}}_i^k$ vectors with many non-zero components.
It may be expedient to build directly into the central problem as varia- bles also some of the original large-size problem's variables, these which have only a few non-zero coefficients in the central constraints

and only zero coefficients in the special sector constraints.[12)]

4) It will be expedient to avoid equalities in the central problem.

Even the constraints which due to their practical contents would actually require the equality form should rather be given as lower bounds.

As a matter of fact, from the viewpoint of the abjective function it may be more advantageous to exceed the lower bound, i.e. in the case of an economic task to produce a surplus than to renounce the inclusion of otherwise advantageous plan proposals because they would make it impossible to satisfy the constraints to equality.

5) Step. z. 5. was originally described as one where the sector defines discrete $[\underline{\underline{u}}_i^k, \underline{\underline{g}}_i^k]$ vector pairs for the sector programs providing the basis of the formulation of new plan proposals.

In addition (or instead), the methods of parametric progra= ming may also be used.

To meet the viewpoints A), B) and C), the central constraints vector $\underline{\underline{u}}_i$ allocated to the sector may be prescribed in parametric

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12)

This was the case with a number of import variables in the course of the economy-wide programing project for 1966-70.

These had non-zero coefficients in two central rows only, namely in the product balance concerned and in the balance of foreign exchange.

Therefore, several import variables have been built also directly and individually into the central problem, and this made the "blending" of the improved program more flexible.

form, and to meet viewpoint D), vector g_i of the objective function coefficients may be prescribed in parametric form.

A single continuous parametric programing may be used for the formulation of a whole series of new plan proposals.

6) Step. z. 4. was originally described as one carried out by the sector independently, using the central information obtained in Step 3. z.

The procedurs may, however, be completed with the following:

The sectors will in every iteration report also the dual solutions of the optimum sector programs used for generating the plan proposals, or, rather, from these dual solutions, the shadow prices of the central constraints.

The centre will compare these , and on the basis of the comparison of the shadow prices of prices of sector programing carried out in the $\{z-1\}$ -th iteration, will prescribe for the z-th iteration the following:

The upper bound of the h-th central resource must be raised (i. e. viewpoint B) must be ensured (in the sectors where the shadow price belonging to the h-th constraint is high.

The bound of the some resource must be decreased) i. e. viewpoint C (must be ensured) in the sectors where the shadow price is low.[13)]

7) In the course of practical application, usually not a single computation is carried out but a whole computation series.

The members of the series may differ from each other both in the objective function and in the value of the individual components of the constraint vector.

When applying the approximation method, it is possible to make preparations for this in advance.

- All objective functions to be amployed in the series are made to figure in the sector programing computations when determining the $[\underline{u}_i, \underline{b}_i, \underline{g}_i]$ -optimal programs.

- Preparations are made for the modification of central constraints in the course of the computation series.

For example, when we know that it is the value of the h-th central constraint that will be increased or decreased in the various

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13) The computational "trick" described in paragraph 6) brigs the basic concept of the decomposition method of fictitious play into the approximation method. (See [3]).

There, the inter-sectoral regrouping of resources is taking place on the basis of the indications of shadow prices obtained in the sectoral programings.

members of the series, we will work out plan proposals with the sectors, of which some use more and some use less of the h-th constraint.

In this way, it will be possible to work out a universal central problem.

This can be employed in the case of any member in the com= putation series, at least in the first iteration, for computing the ini= tial $\underline{\underline{v}}\ \{1\}$ weight vector.

4.2 Change-over to the exact DW-metod

Our method being a variant of the DW-method, we may after any iteration change over from the heuristic approximation method to the exact DW-algorithm.

Let us suppose that after the $\bar{z}$-th iteration, we decided to continue the computation - on the basis of the results obtained up to that point by means of the approximation method - according to the exact algorithm.

Let us write down the $(z+1)$ -th iteration, retaining the numeration of steps as described in section 3.2.

We will now omit to mark the individual steps as "algorith= mic" because this applies now naturally to each step.

Step $(\bar{z}+1)$ .1. (In the centre.)

Let us read now the optimum dual solution of central pro= blem in the $\bar{z}$-th iteration.

Let $\underline{\underline{p}}\ \{z\}$ denote the vector of shadow prices belonging to central constraints (22a).

Step $(\bar{z} + 1)$. 2. Will not be carried out.

Step $(\bar{z} + 1)$. 3. (From centre to sectors).

The transmission of vector $\underline{\underline{p}} \{z\}$.

Step $(z + 1)$. 4. (In the sectors).

Let us formulate the following sector problem:

$$\underline{\underline{B}}_i \underline{\underline{x}}_i = \underline{\underline{b}}_i$$

$$(27) \qquad \underline{\underline{X}}_i \geqq \underline{\underline{0}}$$

$$\left(c_i' - \underline{\underline{p}}' \{z\} \underline{\underline{A}}_i \right) \underline{\underline{x}}_i \longrightarrow \max!$$

Step $(z + 1)$. 5. Let us solve problem (27), and generate, using the solution, the new plan proposal together with the objective function contribution belonging to it.

Step $(\bar{z} + 1$. 6. - $(z + 1)$. 7. These correspond to the identical steps of the z-th iteration described in section 3.2.[14]

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14)

Step. 9 may be complemented with an estimation of the distance from the optimum. The formula is known, from the literature on the subject.

For its description see e.g. J. Stahl's article [9] . Stahl's estimation formula was used in the DW computer program worked out in 1966-67 by the Computing Centre of the Hungarian Academy of Sciences

When putting the approximation method to practical application, the program can be worked out for the electronic computer in a manner that the planners may change over in any iteration from the approximation to the exact method.

Accordingly, the approximation method may also be interpreted as the preparatory phase of the DW-method which provides a suitable initial program for the exact DW-computation.

5 THE PROPERTIES OF THE APPROXIMATION METHOD

5.1 Provable properties

In the following, those properties of the general method described in Chapiter 3 will be dealt with which are susceptible of proof.

First property (Feasibility). The improved programs generated with the general approximation method constitute the feasible solutions of the large-size problem: $\underline{\underline{x}} \{Z\} \in X$.

Proof. First, it must be proven that there is always a feasible solution to the central problem.

This follows trivially from the fact that at least two solutions are known which are per definitionem feasible:

$$(28) \qquad \underline{\underline{y}}_i = y_i^o = 1, \left[y_i^k = 0 \left(k=1,\dots,K_i \{Z\} \right) \right] \quad i=1,\dots,n$$

or:

$$(29) \qquad \underline{\underline{y}} = y^o = 0, \ y^1 = 1, \left[y^k = 0 \ \left(k = 2,\dots,K \{Z\} \right) \right] \quad i=1,\dots,n$$

In the following we must realise that $\underline{\underline{x}} \ \{Z\}$ is the feasi=

ble solution of the large-size problem.

On the one hand, central constraints (22a) of the central pro= blem ensure the satisfaction of central constraints (1a) of the large-size problem, since

$$(30) \qquad \sum_{i=1}^{n} \underline{\underline{T}}_i \{Z\} \; \underline{y}_i \{Z\}^* = \sum_{i=1}^{n} \underline{\underline{A}}_i \underline{x}_i \{Z\} \leq \underline{b}_o .$$

On the other hand, the plan proposal in the central problem are based exclusively on $[\underline{b}_i]$ -feasible programs.

The combination constraints (22b) ensure that the improved program should be composed of the convex combinations of these, i.e. that the special sector constraints (b) of the large- size problem should also be satisfied.

Second property (Improvement). The general approximation method enables the generate a program, the abjective function value of which is definitely higher than that of the comparative program:

$$(31) \qquad \underline{\underline{c}}'\underline{x} \{Z\} > \underline{\underline{c}}'\underline{\underline{x}}^o .$$

Proof. There is certainly known at least one program of which it is obvious that it is more advantageous than the comparative program, and this is the program described in (29).

As a matter of fact, this gives the optimum program of sector problem (14) for every sector with comparative central con= straint allocation $\underline{\underline{U}}^o$.

At the same time, in accordance with assumption 4, the comparative sector programs constitute feasible but non-optimal pro= grams of that problem.

Third property (Monotonity). The objective function value obtained in the z-th iteration is not lower than that obtained in the $\{z-1\}$ -th iteration:

(32) $$\underline{\underline{c}}'\underline{\underline{x}} \ \{z\} \ = \ \underline{\underline{c}}'\underline{\underline{x}} \ \{z-1\} \ .$$

Proof. This follows directly from the fact that - in accordance with Step. z. 7. - the plan proposals accumulated up to the (z-1) -th iteration are not abandoned in the z-th iteration.

The new plan proposal abtained in the z-th iteration will be included only if it improves the value of the objective function.

5. 2. Expectable properties

The method's efficiency will -provided that it is expertly applied - be considerably higher that could be guaranteed on the ba= sis of its mathematically provable properties.

Some non-provable but expectable properties will be descri= bed below.

Here, not only the general method described in Chapiter 3 but also the possibilities of modification and completion autlined in Chapiter 4 will be kept in view.

To illustrate our point, examples of the method's applica= tion in economics and planning will be given.

Our arguments can, however, be extended to application in other fields.

Fourth property (Reality). The simplex-type, finite and exact methods of linear programing, with the DW-method among them, proceed through the extreme points of the convex polyhedron forming the set of feasible programs, leaping from extreme point to extreme point.

In the course of this, we must usually start from absurd programs which do not lend themselves to economic interpretation, with the base containing only the unit vectors.

Then, when the program becomes more interpretable, the objective function value will be still rether disadvantageous.

It is only towards the end of the iteration process that non-absurd programs will be reached which are economically interpretable and sufficiently advantageous from the point of view of the objective function, and the further iterations will then lead up to the optimum.

The proposed approximation method starts from an interior point of the polyhedron[15] and usually also ends at an interior point.

But already the interior point reached in the first iteration will be "sufficiently advantageous".

This is guaranted by the first and second properties : the

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15)

From this point of view, the efficiency of the approximation method ought to be compared - by means of experimental computations - with that of the gradient methods which start also from an interior point.

fact that a sound, more or less rational program based on information from outside the model was included from the outset in the plan pro= posals.

In the further interations, the program's soundness, ratio= nality and economic interpretability will be enhanced by the fact that the plan proposals are not only $[\underline{\underline{b}}_i]$ - feasible, as in the case of the DW-method, but also $[\underline{\underline{u}}_i, \ \underline{\underline{b}}_i]$ -feasible.

In addition, a considerable proportion of the latter is based on evaluable central constraint allocation, which again ensures the realistic character and interpretability of the sector programs.

Information from outside the model will also facilitate the determination of evaluable central constraint allocation.

Fifth property (Continuous improvement). It can be ren= dered probable, although not be proven, that if step z. 4 is skilful= ly carried out, the value of the objective function will not only not dete- riorate from iteration to iteration but considerably improve.

This is based on two economic considerations.

a) Inter-sectoral allocation. In Step. z. 4. new plan propo= sals were worked out which highly economize in the scarce resour= ces and products (see viewpoints B (and D)).

In addition, plan proposals will also be worked out which use more of these resources and products. (See viewpoint) C .)

This affords the possibility to carry out inter-sectoral real= location in the $(z + 1)$ -th iteration.

Should it be advantageous from the point of view of the objective function, the plan proposal economizing to a high degree in a scarce resource or product can be included in one of the sectors and the savings are utilized by the plan proposals of some other sectors which requires more of the scarce resource.

b) Substitution among the factors.

In Step z. 4. plan proposals will also be prepared which use more of the loose constraint (see viewpoint A) and less of the tight constraint (see viewpoint B) and C( ).

Should this be advantageous from the point of view of the objective function, then the new plan proposal which carries out the substitution among the factors will be included in the central program.

Actually, the exact mathematical programing methods, with the exact decomposition methods among them, employ similar eco= nomic principles.

They will do this, however, by observing simultaneously the differential returns (shadow prices) of all resources, factors and products, carrying out usual corrections by using all of them simul= taneously.

(Thus, the DW method carries out the correction of the evaluation of all central constraints simultaneously in the abjective function of the sector computation; the [3] method of fictitious play corrects at the same time all components of the $\underline{\underline{u}}_i$ vector, and so on.)

In the case of the approximation method, on the other hand when working out a new plan proposal (in Step z. 4.), we will mani= pulate only part of the resources, factors and products -probably but some of them - by means of constraints correction or a change in the abjective function, while leaving the rest unchanged.

To mark out the corrections, the exact decomposition me= thods can employ only the information brought into the model in advance or the information which is computed by the iterative process itself in its course (e. g. the shadow prices of the central problem in the DW-procedure, or the shadow prices of the sector problems in the [3] algorithm using fictitious play).

In the case of the approximation method, on the other hand, the palnner will know independently of the model, which resources, factors and products are tight and which ones loose in the large-size problem, and within that in the individual sectors; it is with these in view that he can help intersectoral reallocations and substitutions among factors.

It is partly on the basis of these (and only partly on that of algerithmic central information formed in Step z. 1. of the iteration) that he will decide on the resources, factors and products where cor= rection should be carried out in Step z. 4.

Both inter-sectoral regrouping and substitution among the factors will be facilitated if at least part of the plan proposals is "extremist".

E. g. we have a plan proposal requiring improbably high investment and economizing at the same time highly in manpower; or, conversely, another plan proposal using an improbably high amount of manpower but extremely little investment.

Let us have sectoral plan proposals belonging to central constraint allocations which give extrems preference to some efficient sectors in the allocation of resources to the detriment of all the others. (Such "extremist" programs can easily be generated by choosing suitable $[\underline{\underline{u}}_i, \underline{\underline{g}}_i]$ vector pairs and generating $[\underline{\underline{u}}_i, \underline{\underline{b}}_i, \underline{\underline{g}}_i]$ -optimal plan proposals)

It may be expected that the extremal plan proposals will not receive a weight near unity and may still appear with positive weight in the improved program.

Their existence will facilitate it for the central problem to "blend" in the moot flexible manner the more efficient inter-sectorel allocation, and the best combination of the factors.

When speaking of the fourth property it was laready poin= ted out that a sufficient number of "sound" plan proposals will be needed which are near to the usual allocation.

In addition, however, "extremist", "one-sided" plan propo= sals generated in the above spirit are also needed, to be able to pro= duce the suitable "blend" in the shortest possible way.

Speaking of the improvement of the program, some remarks should be made also concerning the termination of the computation.

This is an inevitably arbitrary decision, in the taking of which the planner will have the rely again to a high degree on infor= mation from autside the model.

On the one hand, he will be able, on the basis of his practical knowledge and experiences, to realise whether $D \{ z \}$, i.e. the improvement against the comparative program, is significant or not, taking into account both the absolute value of the improvement and its magnitude as related to the comparative program as well as its rate in each iteration (whether it is slowing or accelerating, etc.).[16]

On the other hand, he will take into account the "price" that has to be paid for an iteration.

What intellectual and material forces are tied up in carrying out an iteration?

Is it worthwhile to engage the capacity of the planners and the computers in a further iteration, or would it be more reasonable to start instead working on a new problem, and carry out the first iterations of a new plan variant.

Without wishing to lay down a general rule, we may venture to say that within the framework of the approximation method it will hardly be worth while to carry out more than 5 to lo iteration.

By then, the heuristic ideas of the planners will usually be exhausted.

Should we wish to go farther in improving the program,

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16)

It was already mentioned above that the exact DW-method affords the possibility to estimate the distance from the optimum.

In our experience, however, the formula which sets an upper bound to the improvement which can still be achieved, usually underestimates the improvement already achieved and overestimates that which can still be realized.

The application of the formula does, accordingly, not promise much benefit; for the practical planner, the improvement on the comparative program will usually mean more.

then it will be reasonable to change over to the exact method, as de= scribed in section 4.2., taking upon ourselves the cost of the further slow but certain convergence.

Here, we should like to speak of the justification of using the term "approximation method".

In literature, the term is frequently used to denote the al= gorithms which, although not reaching the exact solution of a problem in a finite number of steps, converge to the solution which they may approximate in the case of a suitable number of iteration with <u>any arbitrarily set degree of accuracy.</u> The term "approximation" is, however, not reserved for this; consequently, it is admissible to use it in a looser, to denote a procedure which has no other pro= perties than those described in Chapter 5.

<u>Sixth property (The interpretability of the plan proposal).</u>

In the original DW-method, the meaning of the optimal program is clear.

The plan proposals obtained in the course of the individual iteration have, however, no marked economic meaning, no special characteristic of their own.

In the case of the approximation method, on the other hand it is endeavoured to give each plan proposal some independent econo= mic characteristic, some marked "profile". E.g. "a program eco= nomizing on live labour, with loose investment quota", or "econo= mizing on dollars, to the disadvantage of domestic inputs", etc.

It is in the definition of the "special characteristic" of these proposals that the information material avaitable to the planners from outside the model comes to expression, information that plays again an important role in Step 4.

As a matter of fact, it is on the basis of this information that it will become clear, what modifications in the constraints and what changes in the objective function is it worth while to carry out in order to give the plan proposal a definite "profile".

This will, at the some time, ensure that it is not only the final result - the improved program $\underline{\underline{x}} \{Z\} = \underline{\underline{X}} \{Z\} \underline{\underline{v}} \{Z\}^*$ - that lends itself to analysis.

In each individual iteration, and especially in the last one, the weigths y_i^k will also be significant and suitable for direct economic analysis.

On the basis of what has been said above, let us now summarize the stages where the approximation method makes use of information obtained from outside the model :

a) In including the comparative program in the plan proposals;

b) In determining the evaluable central constraint allocation;

c) in determining vector pairs $[\underline{\underline{u}}_i, \underline{\underline{g}}_i]$ - which provide a basis for generating new plan proposals - in order to facilitate rational intersectoral regrouping and rational substitution between the factors;

d) in forming the "profile" of the plan proposals;

e) in eventuating the additional returns secured as compared with the comparative program, and in terminating the computation.

One of the tasks of research aimed at further developing the method described in this paper will be to work out new ideas and suggestions to accelerate the procedure of improving on the program and to carry out the heuristic steps more efficiently.

5.3 Practical experiences

The approximation method has been used to carry out some minor experimental computation.

In addition it was once applied in practice to a large-size problem within the framework of economy-wide programing for 1966-1970, in Hungary.

The large-size problem contained a total of 2055 constraints and 2424 activity variables (auxiliary variables excluded).

For lack of adequate computing-technical facilities, the exact solution of the problem could not be undertaken.

Any attempt to solve the problem either directly, without decomposition, or by employing one of the decomposition methods would have rendered the computation rather slow.

This would have tied up to computers for a long time, involved high costs and increased the sources of computational errors, endangering thereby the accuracy of the final result.

Therefore, the approximation method described in Chapther 3 and 4 for this paper has been resorted to.

This has enabled the computation of 22 plan variants, i.e. the determination of 22 different improved programs $\underline{\underline{x}}\{Z\}$ obtained with the application of various vector pairs $[\underline{\underline{h}}_o, \underline{\underline{c}}]$.

In the first iteration already, programs were usually reached where other plan proposals beside the comparative $\underset{=}{t}_i^0$ and the sector-optimal $\underset{=}{t}_i^1$ ones appeared with a positive weight.

The improved programs showed a considerable improvement in the value of the objective function as compared with the comparative program.

Let us give some examples :

- The program maximizing consumption ensures a consumption level 5.2 per cent higher than that in the comparative program.

- The program minimizing live labour input saves 6 per cent of the manpower requirements of the comparative program.[17)]

5.4 <u>The justification of employing approximation methods</u>

The discussion of the concrete method outlined in this paper provides an opportunity to make some general remarks on the justification of employing approximation methods in solving economic planning problem.

It is certainly not our intention to put an "ideological complexion" on our difficulties.

Thus, in the economy-wide programing for 1966-1970,

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17) See [6] .

we would have preferred to employ an exact method in solving the concrete problem.

It is not here that the real problem lies but in the following dilemma, well known to all model constructors.

Let us assume that the mathematical character of the model has already been decided upon, and confine us to the case discussed here, that of linear programing.

In that case it is not the size of the model that will be given before hand but (for a particular country, at a given time, or for some definite research team) the computing-technical limitations will be given.

The storage capacity and speed of the available computers, the utilizable machine-time and funds set a limit to the dimension of the linear programing problem that can be solved with the exact method.

The model constructor's dilemma lies in the fact that he must content himself either with this size or - should he want a larger model - with an approximation instead of an exact method.

In both cases a concession is made to tthe detriment of accuracy.

In the second case, this is obvious.

There exist certain constricted views which would take into account this type of inaccuracy only, and judge it one-sidely.

However, the other relationship must not be overlooked either.

The constructing of economic models is in itself an "approximation method".

Every model represents an inaccurate and simplified copy of reality.

The more factors are left out of consideration; the more the possibility of choice is restricted; the higher the degree of aggregation (i.e. the greater the extend to which things are added together which are not directly addible); the leas accurate will be the model, in two senses of the word.

On the one hand, a feasible program of the aggregate model may not be feasible in reality because it fails to satisfy a whole range of constraints which are not set in the model but nevertheless existing in actual fact.

On the other hand, an exactly optimal program of the aggregate model may in reality be sub-optimal, because the realistic alternatives which would have probably appeared in the optimal program of a more detailed model hand not been included in the variables.

The computations based on the model involve two procedures.

First, the infinitely complicated reality is reformulated into a mathematical probelm; then , the mathematical problem is solved.

It is left to the model constructor to chose, in which of the two procedures should he be more accurate to the detriment of the accuracy of the other procedure.

This is a problem of <u>general character</u>, not exclusively related to present-day computing-technical difficulties in Hungary.

If we had computers ten times as large as the present ones, the question would again pose itself: should we content ourselves with the exact solution of the problem which we had formerly been obliged to approach with the approximation method.

Or should we make a step forward in model construction, reflecting reality in greater detail in a larger model (e.g. by replacing single-periodical planning by multi-periodical dynamical planning) but carrying out the computation again on the basis of some approximation method.

No unequivocal and generally valid solution of this dilemma exists.

In pratics , it will be best to follows both paths parallelly, i.e. to construct, on the one hand, models with a higher degree of aggregation for exact computations and, on the other hand, more desaggregate models for approximation methods.

This is exactly what we did when experimenting with the mathematical programing methods to be used in Hungarian five-year planning : exact computations were carried out with a linear programing model of a size of about 80 X 100 [18] and the approximation method was employed in the case of another model with the size of about 2000 X 2500.

The results obtained with the two models can be compared with each other and used for reciprocal control.

Here we have reached an even wider problem of mathematical planning, which should be dealt with only cursorily in this paper, namely the relationshipbetween the planner's living person and the computer.

In the literature on simulation, the term "man-machine"-simulation is widely applied.

It denotes the experiments where part of the operations

18) See [5] .

is carried out by the computer on the basis of an algorithm fixed in advance, while another part is being improvised by the persons taking part in the experiment, who analyse the results obtained in the meantime from the comput er, By analogy in the case of the approximation method we may speack of "machine-man" planning.

This is what the circles and rectangles connected with each other in the schema in Chapter 3 were intended to illustrate.

It must be pointed out that this is not the only case where this applies.

Cooperation of this type between algorithmic, machanized operations and heuristic, intuitive and improvised human intellectual activity is highly characteristic of all mathematical planning.

Even when applying the exact methods, there will be much heuristic thinking and intuition needed; before the computation, in the construction of the model and in the partly subjective estimation of the data; during the computation, in determining the coputation series and sensitivi ty tests to be carried out; and finally, in the evaluation and analysis of the results and in actual decision-taking.

6 ON THE INTERPRETATION OF THE METHOD - CONFLICT AND COMPROMISE

The algorithms of mathematical programing, and especially the decomposition methods can usually be given some economic interpretation which would interpret the procedure as a formal abstract model of planning and of the foundation of decisions.

It is a common characteristic of all interpretations that they do not pretend to formalizing every essential feature of planning and of the foundation of decision.

The various algorithms emphasize usually only one or another element of the process.

The approximation method described in this paper can also be given an economic (and even a general, sociological) interpretation.

When giving this interpretation, we must, naturally detach ourselves from the computing-technical aspect: of the problem.

In this connection, we must not think any longer of the original large-scale problem the solution of which we want to approximate, nor of the fact that the main purpose of generating plan proposals in to advance the improvement of the objective function belonging to the large-scale problem, etc.

The interpretation is the following :

In every organization - be it the state, an administrative unit, some social or political institution, an enterprise, etc. - there exist internal conflicts.

Various parts, sub-units, interest groups will take a stand on the questions of the day on the basis of their own views, real or supposed interests.

Their opinions suggestions and proposals will often contradict each other.

For example, each sub-unit will clain more of the organization's common resources and will want to contribute less.

Even within the sub-unit, the various groups will interpret the specific interests of the sub-unit is various ways.

The collective life of the organization will be possible in spite of these conflicts because some compromise will be made between the contradictory proposals.

When forming the compromise, various criteria may play a part according to how the organization's supreme decision-makers assess the common interest.

Modern sociologists and economists have dealt extensively with the problem of conflict and compromise within the organizations[19] mainly on the basis of empirical observation.

The approximation method - and especially the computation series carried out by means of the approximation method and described in Chapter 4 - may be interpreted as the formalization of the process of working out the conflicting proposals and the compromise made between them.

Conflicts exist on two levels.

On the one hand, the plan proposals compete with one another within the basic unit, the i-th sector.

These can be regarded -due to the fact that each one of them has some marked "profile" of its own - as the expressions of the different views and opinions arising within the sector.

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19)

It is primarily the sociologists engaged in the examination of "formal organization" and the representatives of the so-called "behavourist" school who investigate the problem from this point of view. See [1] and [8] .

Should the sector represent an enterprise, then the plan proposals may reflect the standpoints of the different groups within the firm.[20]

On the other hand, conflicts exist between the sectors, regarding the allocation of the common resources and the carrying of the common burdens and obbligations.

It is the central task of the approximation method to work out a reasonable compromise between the conflicting proposals.

As in real life, here too, the compromise will be formed in iterative process.

First, a temporary pre-compromise will emerge. (First iteration.)

Analysing the weaknesses of this, the decision-makers will ask for further proposals, on the basis of which they will endeavour to reach a more suitable compromise, and so forth.

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20)

For example, within an interprise, the financial department would propose a program simed at increased profits; the sales department would push the production of goods most in demand; the technical development section would urge increased productivity.

Within the latter, one group of engineers would recommend technology "A", another group technology "B", and so forth.

The fact that we are dealing here with an approximation and not with an exact method, does not weaken this interpretation; in fact it renders it rather more realistic.

Such processes do not progress towards an "optimal" compromise based on some strict criterion in actual reality either.

Instead one will content oneself with a "second-best", and "acceptable" final solution.

Instead of strictly enforcing a single optimality criterion, experiments are carried out on the basis of several different viewpoints--which in the case of our formalized procedure corresponds to the fact that hero a series of computations is carried out with the same universal initial central problem but with varying objective functions.

In our opinion, conflict and compromise constitute a particularly important element in the processes of planning and decision-taking.

Our method of approximation is but one of the possible formulations; it will be worth-while to continue research in this direction, to work out and employ also other mathematical models of representing conflicts and compromises.

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