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A MODEL FRAMEWORK FOR ANALYSIS AND SIMULATION OF HOUSING MARKETS

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1. INTRODUCTION

1.1. Empirical background of the research

Our research work has first of all been inspired by the housing difficulties existing in Hungary. Although house building activity is very lively in Hungary, there is a chronic and intensive housing shortage. This is one of the most depressing social problems of the country, permanently occupying the public opinion, the government, and experts in the field.

The main characteristics of the Hungarian housing sector are the following.

Tenement houses with a large number of dwellings are usually *state-owned*. They are maintained by enterprises under the control of local councils. Rents are centrally fixed, and at a very low level: they do not cover even the costs of reproduction and maintenance of the flats. The state housing sector shows a deficit and is heavily subsidized by the state. Rationing of state-owned flats is effected by local councils. Such rationed dwellings are given only to those entitled to them according to certain priority rules based on such criteria as need, merit, (low) social status, queuing time, etc. Because of the multitude of criteria, the dispersion of the time spent in queuing is large; the average queuing time is very long, usually several years.

Another prevalent form of ownership is the *privately owned* dwellings. In a typical case the dwelling is a single-family house inhabited by the owner and his family. Apart from a few administrative restraints, such a house can be sold on a free market at a free price. One version of this form of ownership is the condominium: it is a house consisting of several dwellings each of which is privately owned while the collectively used parts of the building are common property of the owners.

It is possible to sublet privately owned dwellings but only under strict limitations. The most important limitation is that a tenement house consisting of *many* dwellings cannot be private property. Thus, in the privately owned sector, only a family house or a condominium dwelling or a family vacation house can be privately let or sublet. (It is also possible to sublet a state-owned dwelling.) Administrative rules try to fix private rents.

The building costs of family houses and of condominiums are extremely high, particularly as compared to the wages in the formal sector (i.e. the legally accepted sector of the economy). Bank credits advance only a small portion of the costs. Therefore, the family house or condominium dwelling sells at a very high price on the free market. Calculating the imputed monthly rent of such a privately owned dwelling and comparing it with the rent of a state flat of a similar quality, one finds that the former may be four to eight times higher than the latter. This makes it clear that under such circumstances a "grey market" exists in a wide sphere. The tenancy right of state-owned flats is sold, or rather played into the hands of the "buyers", at high prices.

There is no chronic intensive shortage on the market of privately owned houses and dwellings and of private tenements. Solvent demand is sooner or later satisfied at the high prices that are developing. However, the market is functioning with a lot of friction, it is confused and badly organized. Therefore, quite a long search time may be needed until seller and buyer can meet.

There is also a *co-operative* form of building large blocks of dwellings. This is a special form mainly in respect of the organization of the construction work, the selection of membership, and financing; after moving in, the ownership is similar to that of condominium dwellings.

As for quantitative proportions: in towns, and especially in the capital, the prevailing form is the state-owned flat although there are (primarily in the suburbs) quite a number of privately owned dwellings. In the countryside, especially in villages, the single-family house occupied by the owner is the dominant form.

We do not want to go into further details. The Hungarian housing sector has several other features which are changing over time. We think, however, that it also has some features which are not specifically Hungarian but are present also in the housing sectors of numerous other countries. This justifies an examination in terms of a model of more general validity. Apart from the existing Hungarian conditions, *we wish to model a housing sector in which there exist, side by side and intertwined, on the one hand state property, a state rationing system, state price control, and state subsidy of the housing sector, and, on the other hand, private property, free price formation and market trade. Therefore, bureaucracy, shortage, legal, semi-legal and illegal market transactions are all present with* *their typical consequences.* Our research work is aimed at revealing some aspects of the functioning of this symbiosis.

We think that the analysis of this sphere of problems is timely also in Sweden, since, although in different proportions, the above-mentioned symbiosis has existed and to some extent still exists in Sweden.

1.2. Present phase of the research

The sphere of problems outlined in Section 1.1 requires quite extensive research work. One of the authors of this study, Zsuzsa Dániel, is also engaged in research activities in several other, closely related, fields.[1]

The research reported here covers only a small subset of this wide range of problems. It is not *the* housing model we wish to construct, but only *one* of numerous desirable housing models. From the large set of aspects the following are to be stressed.

(1) We wish to grasp the *dynamics* of the allocation. Most models in economics describe allocation in a static form. We waht to bypass this restraint. Both supply and demand are under permanent influence of demographic factors (marriages, birth of children, divorce, children becoming adults, death, and migration) as well as of economic factors (incomes, prices of other goods and services than housing),

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[1] She has made several studies on the following subjects: 1) analysis of housing allocation in Hungary, in particular in regard to its economic and social aspects (cf. [1], [2], [3]), 2) examination of the housing allocation with the aid of "transition matrices" and of Markov models (cf. [4]), and (3) suggestions for a reform of the Hungarian housing sector [5].

etc.). As supply and demand develop over time, so does the allocation and reallocation of dwellings.

(2) In the customary model of the market the buyer makes his choice among goods which are different on account of their technical and other qualities of usage. We deal, of course, also with this kind of choice, but add to it another, more complex aspect of choice. The buyer makes his choice among *alternative "channels" of allocation* of dwellings. He either joins the queue for rationed flats - this is one of the allocation channels through which a dwelling may become available to him - or he starts searching for a single-family house on the private market. That is another allocation channel - and so on.

(3) We wish to examine the effect of both *prices* and *quantity signals*. Within the framework of the present model, the latter are mainly represented by search time and queuing time.

We do not want to construct a model of too great detail. In order to focus on the mentioned three aspects we apply strong simplifications and disregard a number of interdependencies, otherwise important. Let us mention just three aspects which will be disregarded:

(i) Those factors which have an influence on the construction and demolition of houses will not be discussed.

(ii) The effect of income distribution on the demand for dwellings will not be treated. We will consider only one "household type". It is assumed, however, that the behaviour of the various households is differentiated across the aggregate of households. (For example, the alternative allocation channels are chosen in certain definite ratios by the households.)

(iii) Economic growth and, together with it, the growth of incomes, demand, and of housing stock will not be discussed.

The present work is, in a certain sense, the continuation of earlier research work by two of the three authors, János Kornai and Jörgen W. Weibull, on the functioning of markets under circumstances of chronic shortage (cf. [10]). It is mainly the description of the dynamics of the market process that brings this work and the earlier one in a close relationship. At the same time, the present model deviates on some important points from the earlier one. To mention just two: (1) more aspects of demand and supply are endogenous in the present model, (2) unlike the earlier model the present one contains a demographic process.[1]

The present reseach work is closely connected to the problems discussed by one of the authors, János Kornai, in his book "Economics of Shortage" [11] and makes use of the conceptual apparatus and analytical tools set forth in that book.

Now that our subject has been specified as for its *contents*, it remains to clarify which stage the work has reached and the *purpose* of the results reported in the present study. In other words: what is the use of all that is contained in this study?

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[1]Another dynamic model of housing markets, partly inspired by [10], is presented in [12].

(a) We wish to elaborate a *conceptual framework* apt for examination of the above-mentioned three aspects. Parallel with it, we intend to construct a *prototype model* which may serve as a basis for subsequent models, more detailed and comprehensive. It should be mentioned at this point that the present work has been preceded by a few earlier drafts (cf. [6], [7], [8]) with similar objectives. These papers are partly overlapping, partly deviating: they expand or contract various elements of the model. In any case, these preparatory studies may also serve as a basis for further studies.

(b) The present model may serve as a starting point for *computer simulations*. For this purpose the exogenous functions in the model have to be specified and initial values of the variables given. Of course, certain transformations or completions (for example greater disaggregation) may be considered.[1]

(c) The present model may serve as a basis for *theoretical analytical investigations* in which propositions are derived from formalized assumptions. However, it may be that further propositions can be established on the basis of the present assumptions, or on the basis of an extended or modified collection of assumptions.

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[1]One of the earlier versions of the model [8] was used already in a simulation experiment carried out by László Zeöld at the Computing Center of the Hungarian Planning Office.

1.3. Disposition of the report

Section 2 sets forth the model. In the course of the description - while defining and interpreting one or another variable, parameter, or equation - also the economic assumptions implied by the structure of the model are discussed. In addition to this, we also provide formal statements of the assumptions.

At the end of Section 2 we suggest a few numerical indicators which describe the current market conditions.

Section 3 presents two propositions and their proofs.

In Section 4, finally, we point out some further possibilities of the use of the model in analysing housing markets.

2. THE MODEL

2.1. State variables

There are n types of dwelling, n being any positive integer. The classification of dwellings is based on size, quality, equipment, location and other physical properties as well as form of ownership. There is one type of household.

At every point in time, each household can possess at most one dwelling and each dwelling is either occupied (possessed) or vacant.

The state variables are:

$x_{1i}(t)$, the number of households queuing/searching for a dwelling of type i at time t,

$x_{2i}(t)$, the number of households possessing a dwelling of type i at time t,

$x_{3i}(t)$, the number of vacant dwellings of type i at time t.

Thus $x_1(t) = (x_{11}(t),\dots,x_{1n}(t))$ plays the role of (effective) *demand*, and $x_3(t) = (x_{31}(t),\dots,x_{3n}(t))$ that of (effective) *supply*. The *state* of the market at any time t is hence defined by the vector $x(t) =$ $= (x_{11}(t),\dots,x_{1n}(t),x_{21}(t),\dots, x_{2n},x_{31},\dots,x_{3n}(t))$.

Observe that $\Sigma_i(x_{1i}(t)+x_{2i}(t))$ is the total number of households at time t and $x_{2i}(t)+x_{3i}(t)$ is the total number of dwellings of type i at time t.

2.2. Dynamics

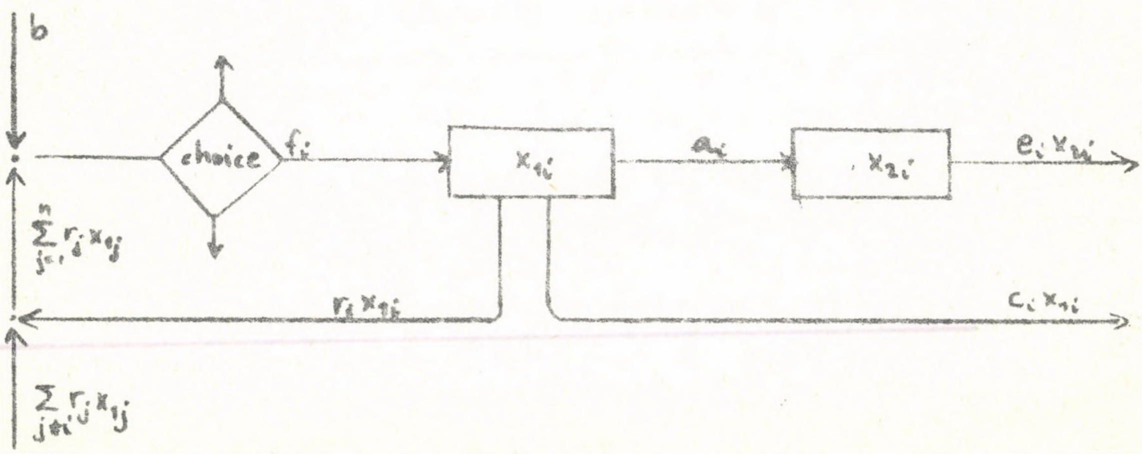
The dynamics of the housing market is here specified in terms of the following system of (generally non-linear) ordinary differential equations:

$$\dot{x}_{1i} = (b + \sum_j r_j x_{1j}) f_i - a_i - (r_i + c_i) x_{1i} \qquad (1)$$

$$\dot{x}_{2i} = a_i - e_i x_{2i} \qquad (2)$$

$$\dot{x}_{3i} = e_i x_{2i} + s_i - a_i - u_i x_{3i} \; . \qquad (3)$$

Here dots signify time derivatives. All state variables and derivatives of state variables are functions of time t, and all parameters $(a_i, b, c_i, e_i, f_i, r_i, s_i, u_i)$ are functions of the present state $x(t)$ and time t. The dynamics defined by these differential equations are illustrated in Figure 1 below.



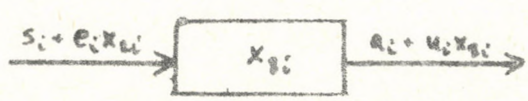


Figure 1: Flow diagram of the model. The upper chart describes the households, the lower the vacant dwellings.

2.3. Interpretations of parameters

Each parameter in equations (1) - (3) represents either a rate (dimension: 1/time), a flow (dimension: units/time) or a flow-share (dimensionless).

Suppressing state variable and time arguments, we have:

a_i, the *trade flow* in allocation channel i . This is the flow of vacant dwellings of type i to households which are queuing or searching for such dwellings.

b , the *birth flow* into the market. This is the inflow of "new" households entering the market e.g. due to immigration or such demographic factors as divorce, growing up of children, etc.

c_i, the *death rate* of households queuing/searching for a dwelling of type i . This is the rate at which queuing/searching households leave the market, e.g. through outmigration and dissolvement of households due to such demographic factors as death of the member of a one-person household or unification of households by marriage.

e_i, the *death rate* of households possessing a dwelling of type i . This is the rate at which possessing households leave the market. The reasons may be the same as for the death rate of queuing/searching households but here also the costs associated with the possession of dwellings may play a more direct role.

f_i, the *choice propensity* to allocation channel i. This is the share of "newly born" (cf. flow b above) as well as of "reconsidering" (see r_i below) households which choose channel i. These choice propensities may e.g. depend on such factors as current prices and queuing times (cf. Section 2.4) etc.

r_i, the *reconsideration rate*. This is the rate at which households queuing/searching for a dwelling of type i reconsider their choice of allocation channel (e.g. due to changing prices or queuing times, cf. Section 2.4).

s_i, the *construction flow*. This is the inflow of "new" vacant dwellings of type i into the housing market (e.g. by construction of new dwellings or remodeling of old dwellings).

u_i, the *demolition rate*. This is the rate at which vacant dwellings of type i are taken out of the housing market (e.g. by demolition or remodeling).

2.4. Signals on the market

In the present model, behaviour is represented in terms of the parameters in equations (1) - (3). These parameters are functions of the present state and time, and may hence reflect influences on behaviour from the current state of the market as well as from exogenous factors. These stimuli are usually conveyed to the economic agents on the market through *signals*. Here we introduce two such signals: *prices* and expected *queuing/search times*, both of which are n-dimensional vectors. Hence, any given state $x(t)$ and time t together determine current prices $p(t) = (p_1(t),\dots,p_n(t))$ and expected queuing/search times $w(t) = (w_1(t),\dots,w_n(t))$, where $p_i(t)$ is the price of a dwelling of type i at time t and $w_i(t)$ the expected queuing/search time for a household to obtain such a dwelling.[1] Since prices and queuing/search times here are functions of the current state and time, dependencies both on internal market conditions and on external factors (such as incomes and prices on other goods and services) may be considered. For example, $p_i(t)$ may be an increasing function of current excess demand $x_{1i}(t) - x_{3i}(t)$ for dwellings of type i, and $w_i(t)$ may be an increasing function of the current queue length $x_{1i}(t)$ divided by the current trade flow $a_i(t)$, etc.

2.5. Indicators of the performance of the market

In analytical studies and computer simulations we want to characterize the performance of the market in different situations. Several relevant indicators can be computed for any given state $x(t)$ and time t. Suppressing time and state variable arguments we have a few immediate examples:

[1] With the acquisition of a new dwelling a *stream* of expenditures arises. In the case of a private house: first the purchasing price, later on the costs of maintenance. In the case of an apartment in a state-owned house: first the semi-legal "key money" (in Hungarian: "lelépési dij"), later on the monthly rent. For the sake of simplicity we do not represent the continuous stream of expenditures in the model. Instead of that we assume the stream is converted in a single "price". (In practice this can be done, e.g. by a "present value" formula.)

$p = (p_1,\ldots,p_n)$ prices

$w = (w_1,\ldots,w_n)$ expected queuing/search times

$x_1 = (x_{11},\ldots,x_{1n})$ number of queuing/searching households (stock)

$x_3 = (x_{31},\ldots,x_{3n})$ number of vacant dwellings (stock)

$s = (s_1,\ldots,s_n)$ production of new dwellings (flow)

$a = (a_1,\ldots,a_n)$ trade (flow)

Here x_1 represents *effective demand* and x_3 *effective supply* and hence

$$z = x_1 - x_3 = (x_{11} - x_{31},\ldots,x_{1n} - x_{3n})$$

reflects (effective) *excess demand* in terms of stocks.

Moreover, if current choice propensities f_i depend on only current prices $p(t)$ and currently expected queuing/search times $w(t)$, then we may readily identify the current *forced substitution ratios*

$$\varphi_i(t) = \frac{f_i(p(t),w(t))}{f_i(p(t),0)} \qquad (i=1,2,\ldots,n) ,$$

i.e. $\varphi_i(t)$ relates the current choice propensity for dwellings of type i to the choice propensity to the same type of dwelling if all expected queuing/search times were negligible. Observe that these forced substitution ratios may exceed unity due to "overspill" from other allocation channels.

In the present model we can, in general, not identify the current number of households living in dwellings of a given type (i.e. the sum of "owner"-households $x_{2i}(t)$ and "sub-tenant"-households in the same type of dwelling). However, we can readily identify the *overall crowding* in the market, i.e. the average number of households per dwelling across all types of dwelling:

$$\xi(t) = \frac{\#\ \text{households}}{\#\ \text{dwellings}} = \frac{\sum_{i=1}^{n} x_{1i}(t) + x_{2i}(t)}{\sum_{i=1}^{n} x_{2i}(t) + x_{3i}(t)}$$

Likewise, for each type of dwelling we may of course identify the demand/supply ratio, i.e. the ratio between, on the one hand, the number of households queing/searching for such dwellings plus the number of households possessing such dwellings, and, on the other hand, the total number of such dwellings:

$$\zeta_i = \frac{x_{1i}(t) + x_{2i}(t)}{x_{2i}(t) + x_{3i}(t)} \quad .$$

Alternatively, one may relate current flows to each other and hence obtain indicators of the direction of change in the market situation.

3. PROPOSITIONS

3.1. Existence and uniqueness of dynamic solutions

For the system of differential equations (1)-(3) to be useful for a general analysis of the dynamics of housing markets, it is necessary to specify conditions on the exogenous data for the existence and uniqueness of its solution. Hence, we will give three conditions which together are sufficient. For convenience we designate $X = R_+^{3n}$ as the *state space* (R_+ denoting the set of nonnegative reals). The relevant *time set* T is supposed to be an interval in R_+. All exogenous functions $a_i, b, c_i, e_i, f_i, r_i, s_i$ and u_i are (real-valued) nonnegative and defined on $X \times T$. With these preliminaries, we are ready to formulate our conditions.

<u>A1</u>. All exogenous functions are continuous and have continuous first partial derivatives with respect to all state variables.

<u>A2</u>. The equation $\sum_i f_i = 1$ holds for all $(x,t) \in X \times T$.

<u>A3</u>. The equation $a_i = 0$ holds for all $(x,t) \in X \times T$ with $x_{1i} = 0$ or $x_{3i} = 0$ (or both).

Condition A1 is a simplification of reality, introduced for technical reasons. Condition A2 simply requires that in every choice situation exactly one of the channels is chosen. Condition A3 requires there to be no trade in a channel in which there is no queing/searching household or vacant dwelling.

Of course, these three conditions do not exclude the possibility that current market conditions (x(t)) and exogenous factors (t) are conveyed via such signals as those discussed in Section 2.4.

Proposition 1: *Under conditions A1 - A3 there exists a unique (local) solution of equations (1) - (3) through every starting point $(x_0, t_0) \in X \times T$. The solution $\{(x(t),t); t \geq t_0\}$ through (x_0, t_0) is a continous trajectory in $X \times T$ (in particular, all state variables remain nonnegative over time).*

Proof: The right-hand side in (1) - (3) is a continuous function of $(x(t),t) \in X \times T$, and it is locally Lipschitzian with respect to $x(t)$. Thus the conditions for the Picard-Lindelöf Theorem are satisfied (cf. e.g. Theorem I.3.1 in [9]). End of proof.

Of course, Proposition 1 is not yet an analytical result offering relevant economic conclusions. It is simply a check on the reasonability of the model. It shows that we have a well-defined model, which describes in a meaningful way how the system evolves over time.

In this general form the model may be applied to a wide range of situations including economic and demographic growth and decline. At every time t, the total number of households is $\Sigma_i(x_{1i}(t) + x_{2i}(t))$. The time derivative of this quantity equals net immigration plus "birth" of households minus "death" of households. Likewise, the total number of dwellings of type i is $x_{2i}(t) + x_{3i}(t)$. The time derivative of this latter quantity equals construction minus demolition (both including remodelling).

Hence, due not only to internal factors but also to time-varying external influences, the housing market may expand, contract or remain constant over time. Below, we investigate conditions for the existence of stationary states of the market.

3.2. Existence of stationary states

A state $x^* \in X$ is said to be *stationary* if $x(0) = x^*$ implies $x(t) = x^*$ for all $t \geq 0$.

In this sub-section we investigate the existence of stationary states under the hypothesis of time-invariant external conditions:

A4. All exogenous functions are independent of time (i.e. $a_i(x,t) = a_i(x,s)$ for all $x \in X$ and $t,s \in T$ etc.).

In other words, condition A4 requires that if the system is in a state x at two different points in time, then all signals, rates, flows and flow-shares are the same at the two occasions. Condition A4 is, of course, a simplification of reality. Nevertheless, it is relevant since it permits analytical studies of properties of the model. In particular, under this condition, the system of differential equations (1) - (3) is *autonomous* and a state x is stationary if and only if $\dot{x} = 0$. In contrast to A4, the following condition seems to be a fairly realistic assumption concerning the circumstances on the housing market.

A5. The functions b, e_i and s_i are bounded. The functions c_i and u_i are bounded from below by positive bounds.

Proposition 2: *Under conditions* A1 - A5 *there exists at least one stationary state.*

Proof: Let $K \subset X$ be defined by

$$K = \{x \in X;\ \sum_{i=1}^{n} (x_{1i} + x_{2i}) \leq \beta/\gamma \ \text{ and } \ x_{3i} \leq (\varepsilon_i \beta/\gamma + \sigma_i)/\kappa_i \ \ \forall i\}$$

where β, ε_i and σ_i are the upper bounds of b, e_i and s_i respectively, γ is the smallest of the lower bounds of the c_i:s, and κ_i is the lower bound of u_i. It is easily verified that K is a positively invariant set under equations (1) - (3), since for any $x \in K$ we have

$$\sum_{i=1}^{n} (\dot{x}_{1i} + \dot{x}_{2i}) \leq \beta - \gamma \sum_{i=1}^{n} (x_{1i} + x_{2i}) \quad \text{and}$$

$$\dot{x}_{3i} \leq \varepsilon_i \beta/\gamma + \sigma_i - \kappa_i x_{3i} \ .$$

Hence, by continuity of the solution of (1) - (3), $x(t_0) \in K$ implies $x(t) \in K$ for all $t \geq t_0$. Moreover, K is a convex and compact subset of X. An application of the Brower Fixed Point Theorem gives the existence of a stationary state in K (cf. e.g. Theorem I.8.2 in [9]). End of proof.

Proposition 2 might be used as a point of departure for economic analysis. The stationary states of the system may be Walrasian or characterized by chronic shortage or excess supply (cf. the different regimes indicated in Section 1.1).

4. FURTHER ANALYTICAL TASKS

The present model or its completed or amended version can be used for the examination of a number of further questions. To mention just a few examples:

- Conditions of stability of the system.
- Identification of different characteristic states of the system ("regimes"). For example, chronic shortage or chronic slack on one or on another market or the same regime on all the markets, or contrary regimes on the different markets, e.g. Walrasian regime on one market simultaneously with a chronic non-Walrasian regime on another market. Investigation of the conditions for the various regimes to become effective.
- Elaboration of the price formation. Studies of the interdependence of prices and "regimes".

In its present form our model is tailored for an examination of the allocation of housing. A question to be considered is whether this type of model can be further developed into a general model of certain allocation processes.

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