PATERNALISM, BUYERS' AND SELLERS' MARKET*

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This paper contributes to the study of non-Walrasian states of the economy and provides a common framework for analysis of excess supply and unemployment in Western economies along with excess demand and chronic shortage in their Eastern counterparts. In particular, the paper formalizes the paternalistic relationship between the state and the firm and examines the comparative implications of state subsidies to firms subject to stochastic economic events. The analysis covers planned, market, and 'mixed' economies and links to some established approaches to disequilibrium phenomena.

Key words: Disequilibrium; planned economy; economic role of the state; stochastic rationing.

1. Introduction

There is a growing interest in the Western literature in the study of non-Walrasian states of the economy. This research programme is often called 'disequilibrium theory' (see, for example, Clower, 1965; Drèze, 1975; Malinvaud, 1977; Grandmont, 1977, and others). In Eastern Europe a similar interest developed (see, for example, Kornai, 1971, 1980). The Western analysts focus their attention on excess supply and unemployment; their Eastern counterparts on excess demand and chronic shortages. After almost two decades of separate work many economists feel that a common theoretical, conceptual, and formal framework is needed for the study of both sets of issues and for the sake of comparative studies. The present paper attempts to contribute to the establishment of such a common analytical framework.

We will contrast our own ideas with the earlier literature in the course of the discussion and in the final section of the paper. Yet we want to draw the reader's attention to two main properties of our model right here in the introduction.

First, we want to go at least one step beyond the formal treatment of the usual economic variables like input, output, demand, supply, price, etc. and include into our analysis some *institutional* phenomena. In particular, we want to formalize the *paternalistic relationship* between the state and the firm. This may appear in a capitalist market economy in the contacts between the government and public and private enterprises. The state makes patronizing interventions and may give support to firms in financial trouble. Similar relationships appear even more intensively between the socialist state and the state-owned enterprise in a planned economy. Paternalism is a manifold, complex social relation: we can capture only one aspect of it, namely state subsidies granted for firms. Paternalism has a deep impact on the actions of the firm.

A second characteristic feature of our approach is the *stochastic* description of economic events. We regard the availability of inputs and the demand for outputs as random variables with fixed probability distributions. With appropriate specifications of these distributions we may describe the degree to which the firm is operating in the environment of a sellers' or a buyers' market. Similarly, subsidies are treated as random variables; their distribution represents the degree of paternalism.

The presentation of the material is organized as follows. First, we introduce a simple model of a firm in a stochastic environment. In Section 3 the firm's prospects for sales, profits, and survival are analysed in the two extreme cases of a pure market and pure planned economy. Section 4 discusses a classification and geometrical illustration of 'mixed' economies. The analysis in Sections 3 and 4 is conditional upon the firm's (effective) demand, while Section 5 introduces demand formation in terms of a satisficing criterion and analyses the influence of paternalism on the firm's demand behaviour. Section 6, finally, relates the present model to some established approaches to disequilibrium phenomena.

2. The model

We elaborate a simple model of a firm operating in a given environment. The firm purchases one input good and produces one output good. It may possess initial stocks of inputs, outputs, and money, and we consider its prospects in a given time period. Hence, the model is static. The environment is stochastic and determines the firm's possibilities of obtaining the input good, selling its output, and receiving subsidies.

The story goes as follows. First, the firm signals its *(effective) demand* x^d for the input good. This good is provided in a market or by a rationing office at an exogenously fixed price w. The firm receives the quantity x of the good, x being a random variable whose probability distribution depends on the quantity demanded. Secondly, the firm chooses what quality q of the output to produce. A high quality enhances the prospects for sales but requires more inputs to produce a given output

quantity. This latter quantity is determined by x and q via the firm's production function f. Having produced $y^s = f(x, q)$ units of output, the firm sells its output at a unit price p (not necessarily exogenously fixed).[1] *Total sales* are represented by a random variable y whose probability distribution depends on y^s, p, and q. The resulting *gross profit* is defined as the random variable $\pi = py - wx$. (Hence, there are no revenues or costs associated with initial holdings of inputs, outputs, or money.) The third and final step during the time period under consideration is the firm's efforts to obtain subsidies. Also, here we employ a stochastic description: the amount of subsidies granted is represented as a random variable r depending on gross profits. The resulting *net profit* is the random variable $\tilde{\pi} = \pi + r$.

The given verbal account will presently be formalized. A central mechanism in this formalization is a *stochastic micro-level version of the short-side rule* (or min-condition as it is sometimes called). This analytically simplifying mechanism is applied at all three stochastic interactions between the firm and its environment. Hence, it may be instructive to first expose this mechanism in general, without reference to the present model.

For this purpose, consider the sales of a good in a shop during a day ω. In the early morning deliveries to the shop arrive and when the shop opens there are $y(\omega)$ units available for sale. Units of the good are sold until stocks are exhausted or the shop closes for the night. Letting $x(\omega)$ be the total number of units requested by the customers and $z(\omega)$ be the number of units sold, we hence have $z(\omega) = \min\{x(\omega), y(\omega)\}$ for all ω. In other words: the short-side rule holds deterministically at the sub-micro level (i.e. every day) while it does not hold at the micro level (i.e. as an average over different days). In general, average sales $E(z) \leq \min\{E(x), E(y)\}$.[2]

Having described our stochastic micro-level short-side rule, we are now in a position to set up the model.

First, let P and Q be subsets of R_+, the set of non-negative reals, and let $p \in P$, $q \in Q$ and $x^d, w \in R_+$. Let $f: R_+^2 \to R_+$ be twice differentiable. Moreover, let $(\Omega, \mathcal{M}, \mu)$ be a probability space, on which are defined non-negative extended-valued random variables (r.v.s) $\bar{x}$, $\bar{y}$, and $\bar{r}$ with probability distribution functions (d.f.s) F, G, and H, respectively.[3] These three r.v.s play the roles of *stochastic rationing variables* for input purchases, output sales, and subsidies.

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[1] Equivalently, one could formally treat both y^s and q as output variables and accordingly write $(y^s, q) \in T(x)$, where $T(x)$ is a transformation curve representing production possibilities associated with input quantity x. However, here the present formalism seems more convenient.

[2] The stochastic short-side rule employed here was used at the macro level in Fair and Jaffee (1972). Muellbauer (1978) applies essentially the same macro-level mechanism, and Kooiman and Kloek (1980) discuss econometric estimation techniques.

[3] I.e. $\bar{x}$, $\bar{y}$, and $\bar{r}$ are $\mathcal{M}$-measurable mappings of Ω into $\bar{R}_+ = [0, +\infty]$ such that $\mu(\{\bar{x} \leq \alpha\}) = F(\alpha) \ \forall \alpha \in \bar{R}_+$, etc. It follows that F, G, and H are non-decreasing and right-continuous mappings of $\bar{R}_+$ into $[0, 1]$ with $F(\infty) = G(\infty) = H(\infty) = 1$.

Secondly, for every outcome $\omega \in \Omega$, let

$$x(\omega) = \min\{x^d, \bar{x}(\omega)\}, \tag{1}$$

$$y^s(\omega) = f(x(\omega), q), \tag{2}$$

$$y(\omega) = \min\{\bar{y}(\omega), y^s(\omega)\}, \tag{3}$$

$$\pi(\omega) = py(\omega) - wx(\omega), \tag{4}$$

$$r(\omega) = \min\{\pi_-(\omega), \bar{r}(\omega)\}, \tag{5}$$

and

$$\tilde{\pi}(\omega) = \pi(\omega) + r(\omega), \tag{6}$$

where π_- is the negative part of π, i.e. $\pi_-(\omega) = \max\{0, -\pi(\omega)\} \; \forall \omega \in \Omega.$[4]

Among these equations, only (5) is not already discussed above. This equation postulates that subsidies are non-negative (i.e. there are not taxes), given only in case of a loss ($\pi(\omega) < 0$), and they never exceed the loss in magnitude.

At this point a comparison with some established models can be made. Clearly, the usual deterministic, neoclassical model of a firm in a perfectly competitive environment falls out as the special case in which p is exogenously fixed, $\bar{x}(\omega) = \bar{y}(\omega) = +\infty$ and $\bar{r}(\omega) = 0 \; \forall \omega \in \Omega$. Moreover, in the special case in which $\bar{x}$ is a constant, equation (1) reduces to the allocation rule in the usual deterministic rationing models (cf. Drèze, 1975, and Benassy, 1975). Finally, it may be noted that the stochastic rationing scheme in Svensson (1980) may be identified as the special case in which the range of $\bar{x}$ contains only two points: one real number and infinity.

In the following analysis, we will assume f and G to satisfy the two conditions:[5]

A1. $f'_x > 0$, $f''_x < 0$, $f'_q < 0$, and $\lim_{x \to \infty} f'_x = 0$.

A2. G has p and q as parameters, and is pointwise non-decreasing in p and non-increasing in q.

In other words: the production function is increasing in input quantity and decreasing in output quality. Marginal productivity with respect to inputs is decreasing and tends to zero as the input quantity tends to infinity. A price reduction or quality improvement either enhances selling prospects or does not affect them. Observe that initial input and output stocks are incorporated as properties of f.[6]

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[4] It follows that all variables are well-defined r.v.s on the given probability space (recalling that f is assumed twice differentiable and hence continuous).

[5] Here f'_x and f'_q denote the first partial derivatives of f with respect to x and q, and f''_x is the second partial derivative of f with respect to x.

[6] Let x_0 and y^s_0 be initial input and output stocks, and suppose $y^s = y^s_0 + g(x_0 + x, q)$ for some 'usual' production function g with $g(0, q) = 0$. Then define $f(x, q) = y^s_0 + g(x_0 + x, q)$.

Having given a formal statement of the model, we are now in a position to derive expressions for the probability distributions of the firm's performance variables x, y, π, and $\tilde{\pi}$. Letting Φ_x, Φ_y, Φ_π, and $\Phi_{\tilde{\pi}}$ denote their d.f.s, we have:[7]

$$\Phi_x(\alpha) = \begin{cases} F(\alpha) & \text{for } \alpha < x^{\text{d}}, \\ 1 & \text{for } \alpha \geq x^{\text{d}}, \end{cases} \tag{7}$$

and

$$\Phi_y(\alpha) = \begin{cases} G(\alpha) + (1 - G(\alpha))F(f_q^{-1}(\alpha)) & \text{for } \alpha < f(x^{\text{d}}, q), \\ 1 & \text{for } \alpha \geq f(x^{\text{d}}, q), \end{cases} \tag{8}$$

where f_q^{-1} denotes the inverse of the production function for a given choice of quality (i.e. $f(f_q^{-1}(\beta), q) = \beta \;\forall \beta$). Furthermore,

$$\Phi_\pi(\alpha) = \mu(\{pf(x, q) - wx \leq \alpha\}) + \int_{[0, x^{\text{d}})} 1_{\{pf(u, q) - wu > \alpha\}} G\left(\frac{wu + \alpha}{p}\right) \mathrm{d}F(u)$$
$$+ \begin{cases} G\left(\dfrac{wx^{\text{d}} + \alpha}{p}\right)(1 - F(x^{\text{d}})) & \text{for } \alpha < pf(x^{\text{d}}, q) - wx^{\text{d}}, \\ 0 & \text{for } \alpha \geq pf(x^{\text{d}}, q) - wx^{\text{d}}, \end{cases} \tag{9}$$

and

$$\Phi_{\tilde{\pi}}(\alpha) = \begin{cases} \displaystyle\int_0^\infty \Phi_\pi(\alpha - u)\,\mathrm{d}H(u) & \text{for } \alpha < 0, \\ \Phi_\pi(\alpha) & \text{for } \alpha \geq 0, \end{cases} \tag{10}$$

where 1_A denotes the indicator function of a set A (i.e. $1_A = 1$ on A and 0 outside A).

In what follows we will study the prospects of the firm in different environments. For this study it is convenient to define an *environment* $\mathscr{E}$ as a quintuple (F, G, H, P, w). Accordingly, we say that the firm's *resource constraint (demand constraint) is harsher* in environment $\mathscr{E}_1$ than in environment $\mathscr{E}_2$ if $F_1 > F_2$ ($G_1 > G_2$). Likewise, we say that the *degree of paternalism* is greater in $\mathscr{E}_1$ than in $\mathscr{E}_2$ if $H_1 < H_2$.[8]

Observe, finally, that the three d.f.s, F, G, and H, to a certain extent represent the macro conditions under which the firm is operating. In this sense, the model explicitly links micro analysis to macro aspects.

3. Pure market and pure planned economies[9]

This section is devoted to a study of two types of environment which are limiting cases in the model presented above. In the first type of environment the firm has

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[7] The straightforward derivations are given in an Appendix.

[8] Evidently, the suggested orderings are only partial: in many cases neither $F_1 > F_2$ nor $F_2 < F_1$ hold, etc.

[9] The discussion in this and the following section disregards some aspects which in many cases are relevant motivations for the public sector, e.g. distributional effects, provision of public goods, and shelter of the buyers against sellers' manipulations.

no difficulties in purchasing inputs at the given price w and there are no possibilities of obtaining subsidies. In the second type of environment the firm has no difficulties in selling its output at the exogenously fixed price p and the firm always gets full subsidies in case of a loss. These two types of environment may be viewed as abstract descriptions of real-life environments in the (non-sheltered) private sector of a market economy and in the state-owned sector of a traditional, pre-reform planned economy, respectively.

More precisely, we call an environment $\mathscr{E}$ an *M1-environment* if $P = R_+$, $F \equiv 0$ on R_+, and $H \equiv 1$. These conditions on F and H can equivalently be written $x = x^d$ (a.e) and $\bar{r} = 0$ (a.e.).

It may be noted that this definition of an M1-environment has as a special case the standard, deterministic model of a firm operating in perfectly competetive markets for all qualities of its output. To see this, observe that the competitive hypothesis postulates the existence of an equilibrium market price $p^*(q)$ for every quality $q \in Q$, such that no output of quality q can be sold at a higher price and any amount of output can be sold at a price not exceeding $p^*(q)$. In terms of the present model:

$$G \equiv \begin{cases} 1 \text{ on } R_+ & \text{for } (p,q) > (p^*(q), q), \\ 0 \text{ on } R_+ & \text{for } (p,q) \leq (p^*(q), q). \end{cases}$$

Accordingly, the firm then takes the price system p^* (a function) for granted, and a choice of x^d and q results in gross profits $\pi = p^*(q)f(x^d, q) - wx^d$ (a.e.). Hence, a perfectly competitive environment is a special kind of M1-environment characterized by a particular type of parametrically discontinuous d.f. G rather than a general d.f. G.[10]

Returning to the more general case of an M1-environment, we readily obtain the following representations of the sales and (net) profit d.f.s (cf. equations (8) and (9)):

$$\Phi_y(\alpha) = \begin{cases} G(\alpha) & \text{for } \alpha < f(x^d, q), \\ 1 & \text{for } \alpha \geq f(x^d, q), \end{cases} \tag{11}$$

and

$$\Phi_{\bar{\pi}}(\alpha) = \begin{cases} G\left(\dfrac{\alpha + wx^d}{p}\right) & \text{for } \alpha < pf(x^d, q) - wx^d, \\ 1 & \text{for } \alpha \geq pf(x^d, q) - wx^d. \end{cases} \tag{12}$$

Before discussing the economic contents of these equations, we set the stage for a pure planned economy, or, more precisely, a *P1-environment* defined as an environment $\mathscr{E}$ in which $\#P = 1$, $G \equiv 0$ on R_+ $\forall q \in Q$, and $H = 0$ on R_+. Alternatively phrased: $P = \{p\}$ for some $p \in R_+$, $y = y^s$ (a.e) and $r = \pi_-$ (a.e). Typically, the price p is determined by a state authority ('the price office') outside the control of the firm.[11]

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[10] In this respect (and only in this respect) the present model resembles random utility models which soften the discontinuity in the standard neoclassical consumer model with respect to utilities.

[11] Of course, in real-life planned economies, some state-owned firms may to some degree influence the setting of its prices.

From equations (8)–(10) we readily obtain:

$$\Phi_y(\alpha) = \begin{cases} F(f_q^{-1}(\alpha)) & \text{for } \alpha < f(x^d, q), \\ 1 & \text{for } \alpha \geq f(x^d, q), \end{cases} \tag{13}$$

and

$$\Phi_{\tilde{\pi}}(\alpha) = \begin{cases} 0 & \text{for } \alpha < 0, \\ \displaystyle\int_{[0,x^d)} 1_{\{pf(u,q) - wu \leq \alpha\}} \, dF(u) & \text{for } 0 \leq \alpha < pf(x^d, q) - wx^d, \\ \displaystyle\int_{[0,x^d)} 1_{\{pf(u,q) - wu \leq \alpha\}} \, dF(u) + 1 - F(x^d) & \\ & \text{for } \alpha \geq pf(x^d, q) - wx^d. \end{cases} \tag{14}$$

Having defined M1- and P1-environments, we are now in a position to analyze the implications of the firm's choice of x^d, q, and p for the prospects for sales and net profits (recalling that in the P-environment the firm has only one 'choice' of p).

For this purpose, it is convenient to say that sales (or net-profit) prospects are *uniformly improving* (*deteriorating*) with respect to a given decision variable if Φ_y (or $\Phi_{\tilde{\pi}}$) is pointwise non-increasing (non-decreasing) in the decision variable. If a prospect is uniformly improving or deteriorating with respect to some decision variable, then it is said to be *monotonic* in the variable.

Observation 1. *In both types of environment, sales prospects are uniformly improving in x^d. In an M1-environment, they are uniformly deteriorating in p and not necessarily monotonic in q. In a P1-environment, they are uniformly deteriorating in q.*

(See Appendix for proofs of this and subsequent observations.)

Next, we consider the prospects of (net) profits. Let $\hat{x}(p, q)$ denote the unique value of x at which the potential-profit function $v(x) = pf(x, q) - wx$ achieves its maximum for given values of p and q.[12]

Observation 2. *In an M1-environment, (net) profit prospects are not necessarily monotonic in x^d for $x^d < \hat{x}(p, q)$ and uniformly deteriorating for $x^d > \hat{x}(p, q)$. Moreover, they are not necessarily monotonic in p and q. In a P1-environment, net-profit prospects are uniformly improving in x^d for $x^d < \hat{x}(p, q)$, uniformly deteriorating in x^d for $x^d > \hat{x}(p, q)$, and uniformly deteriorating in q.*

What about survival perspectives in the two environments? Here a striking difference between real-life market economies and planned economies appear: while survival is uncertain for many firms in the private sector of a market economy, it is more or less guaranteed for firms in the state-owned sector of a planned economy.[13]

[12] The function $v: R_+ \rightarrow R$ is continuous and strictly concave by A1.

[13] See Chapters 13 and 22 in Kornai (1980).

In general, the survival of a firm (or at least of its management) depends on many factors, including its performance in production, sales, and (net) profits. Below, we study the implications of a certain simple survival criterion in the present model. This criterion requires the firm to make neither a *sales failure* nor a *net-profit failure* in the sense that y and $\tilde{\pi}$ should exceed some predetermined thresholds δ_1 and δ_2, respectively. The firm *survives* if and only if both of these conditions are satisfied.

Observe that δ_1 is also an indirect requirement on production and in a P1-environment it reduces to a requirement on production y^s only. Likewise, the (net-) profit condition may alternatively be viewed as a condition on final money holdings since if initial money holdings are m_0, then final money holdings are $\tilde{\pi} + m_0$.

For simplicity, we assume $\delta_2 \le 0$, and examine the two aspects of survival separately:

Observation 3. *In both environments a sales failure is certain for (x^d, q) such that $f(x^d, q) \le \delta_1$. For other choices of (x^d, q), the probability of a sales failure is independent of x^d and non-increasing (non-decreasing) in q in an M1-environment (P1-environment). In an M1-environment, a profit failure is certain for (x^d, q) such that $pf(x^d, q) - wx^d \le \delta_2$. For other choices of (x^d, q) it is non-decreasing in x^d and non-increasing in q. In a P1-environment, (net) profit failures are excluded.*

Hence, the survival motive suggests a moderate demand for inputs and a high quality of the output good in an M1-environment. In a P1-environment on the other hand, the same motive suggests only a lower bound on input demand and a low quality of the output. Hence, in a P1-environment the survival motive puts no brake on the 'expansion drive' and, as a consequence, demand for inputs becomes almost insatiable.[14]

A summary of qualitative observations concerning M1- and P1-environments is given in Table 1, where 'improving' signifies 'uniformly improving prospects', 'deteriorating' stands for 'uniformly deteriorating prospects', and 'ambiguous' signifies 'not necessarily monotonic'. The slashes in the second row separate the regions $x^d \le \hat{x}(p, q)$ and $x^d > \hat{x}(p, q)$.

Table 1

	x^d		q	
	M1-env.	P1-env.	M1-env.	P1-env.
Sales	improving	improving	ambiguous	deteriorating
Net profits	amb./deter.	imp./deter.	ambiguous	deteriorating
Survival	deteriorating	improving	improving	deteriorating

Finally, we offer a brief comment concerning the dependency of the firm's pros-

[14]Cf. Chapters 3, 4, and 9 in Kornai (1980).

pects on its exogenous characteristics. In the present model the firm is characterized only by its production function f (incorporating initial input and output stocks), and it follows immediately from equations (11)–(14) that all prospects, including survival, are uniformly improving in f in both environments (in the canonical ordering of functions f). However, in real-life planned economies, also the firm's personal connections with the state authorities are of fundamental importance. Hence, a firm with good connections in the allocation channel for inputs has a favourable d.f. F and a firm with good connections to the subsidy-granting authorities has a favourable d.f. H.[15]

4. 'Mixed' economic systems

In the preceding section we discussed in some detail two environments which are limiting cases and which may serve as points of reference in comparisons of the (non-sheltered) private sector in market economies with the state-owned sector in planned economies. Here we extend the discussion to other sectors of these two types of economies by means of a classification scheme and a geometrical illustration.

In a market economy we distinguish four types of environment. A common feature of all four environments is that there are no difficulties in obtaining the input good at the given price w, i.e. $F = 0$ on R_+. In the *non-sheltered private* sector (=M1-environment) there are no possibilities of receiving subsidies ($H = 1$), while in the *sheltered private* sector there are such possibilities ($0 < H < 1$). Following Johnston (1975) and Bacon and Eltis (1976) we may usefully divide the public sector into one part which produces marketed output (at market prices) and another which produces non-marketed output (at exogenously fixed, low prices).[16] In both subsectors subsidies are granted ($H = 0$ on R_+), the difference being that in the *marketed public* sector sales conditions are similar to those for private firms, while in the *non-marketed public* sector sales conditions are similar to those for firms in a P-environment: $G = 0$ for all $q \in Q$ at the given price p.

Likewise, environments in a planned economy may be divided into four groups. In the first three, i.e. the *state-owned* (=P1-environment), *cooperative,* and *regulated private sectors,* all output can be sold at the administratively regulated price: $G = 0$ on R_+ for all $q \in Q$. Subsidies are granted in the state-owned sector ($H = 0$ on R_+), while they are sometimes given to cooperatives ($0 < H < 1$) and virtually never to private firms ($H = 1$). In the fourth sector, the *non-regulated private* sector, prices are more or less free and conditions for selling and receiving subsidies are similar to those in the non-sheltered private sector of a market economy. Table 2 shows the eight types of environment.

[15] More precisely, F_1 (H_1) is *more favourable* than F_2 (H_2) if $F_1 < F_2$ ($H_1 < H_2$).

[16] In fact, Johnston and Bacon–Eltis suggest the division of the whole economy into two sectors only: one which produces marketed output (including the private sector and parts of the public sector) and another which produces non-marketed output (a part of the public sector).

Table 2

	F	G	H	Price
Market economy:				
non-sheltered private (M1)	0		1	flexible
sheltered private (M2)	0		(0, 1)	flexible
marketed output, public (M3)	0		0	flexible
non-marketed output, public (M4)	0	0	0	fixed
Planned economy:				
state-owned (P1)		0	0	fixed
cooperative (P2)		0	(0, 1)	fixed
regulated private (P3)		0	1	fixed
non-regulated private (P4)			1	flexible

Next, we give a geometrical illustration of the three environmental aspects, F, G and H, from the viewpoint of a firm that has chosen its output price p and quality q. More precisely, we consider a parametric family of triplets (F_a, G_b, H_c) where each of the three parameters range over the unit interval. We suppose that F_a is uniformly increasing in a with $F_0 = 0$ on R_+ and $F_1 = 1$. The parameter a is accordingly called the *resource-constraint parameter*. Likewise, G_b is uniformly increasing in b with $G_0 = 0$ on R_+ and $G_1 = 1$, and b is hence called the *demand-constraint parameter*. Finally, H_c is uniformly decreasing in c with $H_0 = 1$ and $H_1 = 0$ on R_+, c being referred to as the *paternalism parameter*. In sum: each environment in the family is represented as a point (a, b, c) in the three-dimensional unit cube, see Fig. 1.
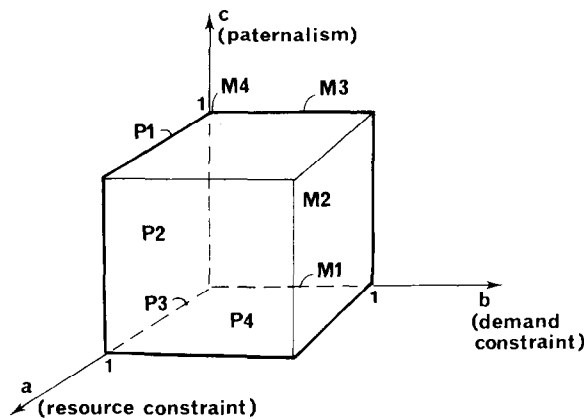

Fig. 1. Parametric representation of environments.

In the notation of Table 2, we evidently have each of the four types of environments M1, M3, P1, and P3 as an edge of the cube. Moreover, environments of type M2 and P2 are points on the surfaces spanned by M1 and M3 and P1 and P3, respectively. Environment M4 is the corner at the intersection of edges P1 and M3, and environments of type P4, finally, are points on the bottom surface, spanned by edges M1 and P3. The origin in this coordinate system is the traditional, deterministic environment of perfect competition: there the firm experiences no quantity constraints on buying or selling and subsidies are excluded.

In the preceding section the firm's prospects at the edges M1 and P1 were studied in some detail and qualitative differences were observed. Hence, qualitative differences in the behaviour of firms can be expected. An interesting question then is the influence of the degree of paternalism on the behaviour of firms. In particular: What are the effects of the degree of paternalism on the firm's effective demand x^d? Clearly, an analysis of this and related questions requires the introduction of a behavioural hypothesis concerning the firm's decision-making.

5. Paternalism and demand behaviour

Here we consider the influence of paternalism on our firm's effective demand x^d for its input good. In this context, the prices p and w, as well as the output quality q, are assumed to be fixed. The firm's demand behaviour will be described in terms of a satisficing rule (cf. Simon, 1959), and two types of environment – demand constrained and resource constrained – will be considered. For simplicity, the firm is assumed to know its environment as represented by the d.f.s F, G and H. We focus on the effect of paternalism only, the dependency on other factors, such as prices and buying and selling conditions, will not be considered.

More precisely, for given d.f.s F and G we assume there to exist $\alpha, \beta \geq 0$ and $0 \leq \varepsilon \leq 1$ such that for every degree of paternalism H, the firm's choice of x^d satisfies the three constraints $E(\pi_+ \mid x^d) \geq \alpha$, $E(y \mid x^d) \geq \beta$, and $P(\tilde{\pi} \geq 0 \mid x^d) \geq 1 - \varepsilon$, where π_+ is the positive part of π, i.e. $\pi_+ = \max(0, \pi)$. Accordingly, we refer to the behavioural parameters α and β as the firm's *aspiration levels* for profits and sales, and to ε as the firm's *risk threshold*. The interpretation of $P(\tilde{\pi} \geq 0 \mid x^d)$ is the *survival probability*, $\tilde{\pi} < 0$ representing the event of a loss in spite of subsidies. Let X^d denote the set of points $x^d \in R_+$ satisfying the three satisficing constraints. We assume α, β, and ε to be such that X^d is non-empty.

As for environments, our concern is for environments on the 'M side' and 'P side' of the parametric cube in Fig. 1. However, since our analysis does not require a parametric representation, we more generally define an *M-environment* as an environment in which $F = 0$ on R_+ (no resource constraint). Clearly, M1, M2, M3, and M4 environments are examples of M-environments. Symmetrically we define a *P-environment* as an environment in which $G = 0$ on R_+ (no demand constraint). Evidently, P1, P2, and P3 are examples of P-environments, while P4 is neither an

M- nor a P-environment. For clarity of exposition, we will restrict our studies to continuous d.f.s F, G, and H.

As before, let $\hat{x}$ denote the unique value of x at which the potential-profit function $v(x) = pf(x, q) - wx$ achieves its maximum, and let $x_0 = \sup\{x \geq 0; v(x) \geq 0\}$. Hence, $\hat{x}$ is the usual Walrasian demand, and $0 \leq \hat{x} < x_0 < +\infty$ by A1.

First, we consider the case of an M-environment. From equation (12) we readily obtain:

$$E(\pi_+ \mid x^d) = \int_0^\infty [1 - \emptyset_\pi(u)] \, du = \int_0^{pf(x^d, q) - wx^d} \left[1 - G\left(\frac{u + wx^d}{p}\right)\right] du \quad (15)$$

for $x^d < x_0$, and $E(\pi_+ \mid x^d) = 0$ for $x^d \geq x_0$. Clearly $E(\pi_+ \mid x^d)$ is a continuous function of x^d, bounded by $v(\hat{x})$, achieving its maximum somewhere in the interval $[0, \hat{x}]$, and decreasing on $(\hat{x}, x_0)$. Likewise, from equation (11) we have

$$E(y \mid x^d) = \int_0^\infty [1 - \emptyset_y(u)] \, du = \int_0^{f(x^d, q)} [1 - G(u)] \, du. \quad (16)$$

Clearly $E(y \mid x^d)$ is a continuous and non-decreasing function of x^d. Finally, by equations (10) and (12) we obtain for $x^d < x_0$:

$$P(\tilde{\pi} \geq 0 \mid x^d) = 1 - \int_0^\infty G\left(\frac{wx^d - u}{p}\right) dH(u). \quad (17)$$

Hence, $P(\tilde{\pi} \geq 0 \mid x^d)$ is a continuous and non-increasing function of x^d with $P(\tilde{\pi} \geq 0 \mid 0) = 1$.

Let ξ_M^d be the *effective-demand correspondence* carrying H to X^d. With self-explanatory notation we readily obtain:[17]

Observation 4. *The effective-demand correspondence in an M-environment is a non-decreasing function of the degree of paternalism, i.e. $H_1 > H_2 \Rightarrow \xi_M^d(H_1) \leq \xi_M^d(H_2)$.*

Observe that in the special case of a perfectly competitive market for the output good ($G = 0$ on R_+) and maximal profit aspirations ($\alpha = v(\hat{x})$, $\beta = 0$, and $\varepsilon > 0$) we obtain $\xi_M^d(H) = \{\hat{x}\}$ for every d.f. H, i.e. the firm's effective demand correspondence then coincides with the usual neoclassical demand function. Phrased differently: with any degree of paternalism but in the absence of a demand constraint, the purely profit-oriented firm behaves exactly like the standard firm in the Walrasian model.

Turing to the case of a P-environment, we first note that $E(\pi_+ \mid x^d)$ is a continuous function bounded by $v(\hat{x})$ and achieving its maximum at $x^d = \hat{x}$. This last observation is intuitively clear since the firm perceives no demand constraint on sales,

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[17] As before, we write $H_1 > H_2$ if $H_1(\alpha) > H_2(\alpha)$ $\forall \alpha \in (0, \infty)$. Moreover, for two sets $A, B \subset R$, we write $A \leq B$ if there are $a \in A$ and $b \in B$ such that $a \leq B$ and $b \geq A$.

and the rationing scheme is non-manipulable at the sub-micro level and hence it does not pay off to ask for more than $\hat{x}$.[18] Concerning expected sales, we note that

$$E(y \mid x^d) = \int_0^\infty [1 - \Phi_y(u)] \, du = \int_0^{f(x^d, q)} [1 - F(f_q^{-1}(u))] \, du \quad \forall x \geq 0, \quad (18)$$

by equation (13). Clearly $E(y \mid x^d)$ is a continuous and non-decreasing function x. Finally,

$$P(\tilde{\pi} \geq 0 \mid x^d) = \begin{cases} 1 & \text{for } x \leq x_0, \\ 1 - \int_0^{wx^d - f(x^d, q)} [1 - F(v^{-1}(-u))] \, dH(u) & \text{for } x > x_0, \end{cases} \quad (19)$$

where $v^{-1} : (-\infty, 0) \to (x_0, \infty)$ is the (restricted) inverse of the potential-profit function. Clearly $P(\tilde{\pi} \geq 0 \mid x^d)$ is a continuous and non-increasing function of x^d. Hence,

Observation 5. *The effective-demand correspondence in a P-environment is a non-decreasing function of the degree of peternalism, i.e. $H_1 > H_2 \Rightarrow \xi_P^d(H_1) \leq \xi_P^d(H_2)$.*[19]

In symmetry with the case of an M-environment we obviously again obtain the usual Walrasian demand function in the special case of no resource constraint ($F = 0$ on R_+) and maximal profit aspirations ($\alpha = v(\hat{x})$, $\beta = 0$, and $\varepsilon > 0$).

As a final comment concerning the influence of paternalism on firms' behaviour, we would like to relate to the discussion of Keynesian and classical unemployment in Malinvaud (1977).

For this purpose, we consider the event of being rationed in an arbitrary environment. The firm is rationed in its purchase of the input good if $\bar{x} < x^d$, and it experiences rationing on sales if $\bar{y} < y^s$. Accordingly, if we take the input good to be labour, then we may say that the firm operates in *Keynesian unemployment* if $x^d < \bar{x}$ and $\bar{y} < y^s$, and in *classical unemployment* if $x^d < \bar{x}$ and $\bar{y} \geq y^s$. The probability of being in either of these two types of unemployment evidently is $1 - F(x^d)$, while the probability of being in Keynesian unemployment is $(1 - F(x^d))G(f(x^d, q))$, granted that G is continuous. Accordingly, the conditional probability of being in Keynesian unempolyment is $G(f(x^d, q))$.

An interesting question then is what influence the degree of paternalism has on these probabilities, the influence being transmitted from the d.f. H via the firm's choice of x^d, q, and p. For example, if we assume – as the preceding analysis suggests – that a higher degree of paternalism leads to a larger effective demand x^d, then a higher degree of paternalism implies an increase in the conditional probability of Keynesian unempolyment, *ceteris paribus*. However, if the higher degree of

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[18] For a definition and discussion of 'manipulable' and 'non-manipulable' rationing schemes, see Benassy (1982). For a similar argument in the case of stochastic rationing of a consumer, see Benassy (1982, Appendix B).

[19] Cf. also Chapters 3, 6, and 9 in Kornai (1980).

paternalism induces an increase also in other firms' demand for our firm's output, then the tendency towards Keynesian rather than classical unemployment is at least partly counteracted, and the net effect is ambiguous.

6. Comparison with the standard theoretical framework

Arriving at the end of our analysis, we make a brief comparison with the well-known standard neoclassical and disequilibrium frameworks. Instead of pointing to one or another specific paper, we will keep in mind an abstract prototype of this family of models.

(1) The most important distinction: we want to build into our framework institutional factors, explaining chronic deviations from Walrasian equilibrium and not only the usual economic variables like price or wage, etc. As an example of this effort, we consider the degree of paternalism as an explanatory variable. The more a firm can expect to be bailed out by the government in case of long-lasting troubles, the more it will be inclined to expansion and, as a consequence, to exhibit almost insatiable demand. The existence of a modern welfare state, and − even more − the existence of a socialist state as owner of all big enterprises, creates new conditions for the behaviour of firms. Paternalism is, of course, only an example of the more general effort to reflect the impact of institutional factors in formal socio-economic models. Such factors are missing from most standard models which explains disequilibrium mainly by the impact of 'economic policy' and price rigidities.

(2) In describing the behaviour of the firm, we want to have a more general framework than the usual profit-maximizing pattern. We believe, together with many other economists and sociologists, that every organization, including firms, has several (often conflicting) goals. We incorporate three of them in our model: quantitative growth, profit, and, as an ultimate objective, survival. Firms in different social environments can be characterized by different constellations of these (and other) motivations. In addition, we apply − following Simon (1959) − the satisficing model of decision-making. This approach seems to be more general and realistic, and in the present model profit maximizing appears as a special case of the more general pattern.

(3) Most work in the Walrasian and disequilibrium schools applies deterministic descriptions of systems. The deterministic short-side rule in the standard disequilibrium model assumes that the firm is either constrained on the input side or not, and the same 'yes-or-no' description holds for the output side as well. We think that this is not a 'yes-or-no' issue but a matter of degree.[20] The firm's environment can be a buyer's market or a seller's market with stronger or weaker intensity and, accordingly, with stronger or weaker impact on the firm's decisions and actions. The stochastic framework applied in this paper provides an apparatus to reflect the

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[20]For references to other stochastic versions of the short-side rule, see footnote 2.

intensity of the external macro-conditions on the activity of the micro-unit, i.e. the firm.[21]

The same reasoning applies to the stochastic representation of state subsidies. Again, paternalism can be more or less intense and our formal apparatus may reflect the certainty or uncertainty in the firm's expectations concerning the paternalistic interventions of the state.

(4) Most standard models deal only with the control of the quantity of output. We find it necessary to include the control of quality. All the more since one of the most serious consequences of a chronic shortage situation is the neglect of quality in favour of forced expansion and a drive for more quantity. In the present paper we are far from exploiting the possibility of including quality as an endogenous variable but at least we show that here are some further opportunities for analysis.

We are aware that the aims outlined above are ambitious and the results we achieved up to now are rather modest. Our primary aim with this study was to pose a certain set of *questions* and also a *framework* for analysing these issues. There is no novelty in our *answers:* we are satisfied seeing that in this novel framework we are able to reproduce theoretical propositions known from the literature. There are several roads leading to further extensions. Perhaps the most important direction of research will be the study of systems with several firms. Each firm making up part of other firms' environments, one may ask about the compatibility between behaviour and environments. Another intriguing but difficult question is the question of survival and 'natural selection' of firms or firms' decision rules (cf. Winter, 1964).

Acknowledgements

[21] Cf. also the stochastic approach to degrees of imbalance or 'tension' in aggregate markets in Malinvaud (1981).

Appendix

Derivation of equations (8)–(10)

$$\phi_y(\alpha) = \mu(y \le \alpha) = E[\mu(y \le \alpha \mid x)]$$
$$= E[\mu(\min\{\bar{y}, y^s\} \le \alpha \mid x)] = E[1_{\{y^s \le \alpha\}} + 1_{\{y^s > \alpha\}} G(\alpha)]$$
$$= \mu(f(x,q) \le \alpha) + \mu(f(x,q) > \alpha) G(\alpha).$$

$$\phi_\pi(\alpha) = \mu(\pi \le \alpha) = E\left[\mu\left(y \le \frac{wx+\alpha}{p} \,\middle|\, x\right)\right]$$

$$= E\left[1_{\{f(x,q) \le (wx+\alpha)/p\}} + 1_{\{f(x,q) > (wu+\alpha)/p\}} G\left(\frac{wx+\alpha}{p}\right)\right]$$

$$= \mu\left(f(x,q) \le \frac{wx+\alpha}{p}\right) + \int_{[0,x^d)} 1_{\{f(u,q) > (wu+\alpha)/p\}} G\left(\frac{wu+\alpha}{p}\right) dF(u)$$

$$+ 1_{\{f(x^d,q) > (wx^d+\alpha)/p\}} G\left(\frac{wx^d+\alpha}{p}\right)(1 - F(x^d)).$$

$$\phi_{\tilde{\pi}}(\alpha) = \mu(\tilde{\pi} \le \alpha) = \mu(\pi \le \alpha - \bar{r}) \quad \text{for } \alpha < 0,$$
$$\text{because then } [\tilde{\pi} \le \alpha] \Leftrightarrow [\pi + \min(\pi_-, \bar{r}) \le \alpha]$$
$$\Leftrightarrow [\pi \le 0 \text{ and } \min(0, \bar{r}+\pi) \le \alpha] \Leftrightarrow [\pi \le 0 \text{ and } \bar{r}+\pi \le \alpha]$$
$$\Leftrightarrow [\pi \le 0 \text{ and } \pi \le \alpha - \bar{r}] \Leftrightarrow [\pi \le \alpha - \bar{r}].$$

Proof of Observations 1–5

Observation 1. Apply A1 and A2 to equations (11) and (13).

Observation 2. In the case of an M1-environment, apply A1 and A2 to equation (12). In the case of a P1-environment, observe that for $\alpha > 0$, $\phi_{\tilde{\pi}}(\alpha) = \mu(v(x) \le \alpha)$. Since v is concave with $v(0) = 0$ and $v(\infty) < 0$, there are $x_1(\alpha)$, $x_2(\alpha)$ such that $0 < x_1(\alpha) \le \hat{x}(p,q) \le x_2(\alpha) < \infty$, and $[v(x) \le \alpha] \Leftrightarrow [x \le x_1(\alpha) \text{ or } x \ge x_2(\alpha)]$. Hence, by equation (7):

$$\phi_{\tilde{\pi}}(\alpha) = \mu(x \le x_1(\alpha) \quad \text{or} \quad x \ge x_2(\alpha))$$
$$= \begin{cases} 1 & \text{if } x^d \le x_1(\alpha), \\ F(x_1(\alpha)) & \text{if } x_1(\alpha) < x^d < x_2(\alpha), \\ F(x_1(\alpha)) + 1 - F(x_2(\alpha)) & \text{if } x^d \ge x_2(\alpha). \end{cases}$$

Clearly, $x_1(\alpha)$ is increasing in α and $x_2(\alpha)$ decreasing.

Observation 3. The proof follows from equations (11)–(14).

Observation 4. Let $X_1^d = \{x \geq 0; E(\pi_+ \mid x) \geq \alpha\}$, $X_2^d = \{x \geq 0; E(y \mid x) \geq \beta\}$, and $X_3^d = \{x \geq 0; P(\tilde{\pi} \geq 0 \mid x) \geq \varepsilon\}$. Then $X^d = \bigcap_{i=1}^3 X_i^d$. Clearly, X_1^d and X_2^d are functionally independent of H, while $X_3^a = [0, x(H)]$, where $x(H_1) \leq x(H_2)$.

Observation 5. To establish equation (19), note that $\phi_{\tilde{\pi}}(0) = \int_0^\infty \phi_\pi(-u) \, dH(u)$ by equation (10), and that for $x > x_0$ and $u \geq 0$:

$$\phi_\pi(-u) = \begin{cases} 1 - F(v^{-1}(-u)) & \text{for } u \leq -pf(x^d, q) + wx^d, \\ 0 & \text{otherwise.} \end{cases}$$

As in the proof of Observation 4, X_1^d and X_2^d are functionally independent of H, while $X_3^d = [0, x(H)]$, where $x(H_1) \leq x(H_2)$.

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