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THE DETERMINATION OF THE OPTIMAL INVESTMENT PLAN FOR AN INDUSTRIAL SECTOR BY THE USE OF LINEAR PROGRAMMING

Results of Calculations Made in the Textile Industry

Research was recently completed on the mathematical calculation of the program of optimal investment and technological development during the period of the Second Five Year Plan.

The programming was preceded by extensive research. We worked out the methodology of the calculations, first trying it out within the dimensions of a single plant, the Goldberger Weaving Mill. (1) Results of this experiment were discussed in detail by economists and textile engineers, and only thereafter were calculations made for the industry as a whole. Many experts took part in gathering data and setting up the model calculations. (2) Finally the most important numerical calculations were made on an M-3 type electronic computer in the Calculations Technique Center of the Hungarian Academy of Sciences. (3)

Basic Features of the Model

For the purpose of economic analysis we used seventeen types of models in all. In basic concept they are similar, and differ only in one or another detail. In this article I discuss only their most essential traits, avoiding, so far as possible, the special technical problems of the textile industry and the mathematical aspects of the models. A more detailed description will be found in an appendix to this article.

1. The sphere of programming. Our calculations embraced all the plants of the Textile Industry Management, but among them we considered only the weaving mills.

We divided the weaving machine stock into three main categories: narrow-smooth machines, broad-smooth machines, and fancy-weft machines. These three categories include 97 per cent of the present weaving machine stock. The remaining 3 per cent comprises special weaving machines (jacquard, terry cloth, etc.); these we have omitted from our calculations.

To the weaving mills also belong — from both a technological and organizational standpoint — machines for pre-weaving operations. Among preparatory phases we dealt with the four most important: cross-spooling, weft-spooling, upcasting and sizing. We disregarded minor operations requiring far less investment (such as tying, for example).

2. Problems of selection. In defining the investment plan we had a choice from among a number of possible alternatives. It is characteristic of programming that it responds to a whole series of selection problems simultaneously. We constructed our model in such a fashion that it would answer the following questions, widely debated in the weaving industry:

Should we renovate existing factories or build new ones?

Some of the present older machines can be rebuilt or modernized at greater or less cost. What should be done about these? Should we keep them in their present form, modernize them or dismantle them?

Other existing old machinery offers no worthwhile possibility of renovation. So there are but two choices: should we keep them or dismantle them?

In acquiring new machines we can select among many new types. Which types should we choose, and how many of each type should we purchase?

The program determined by the calculations prescribes the investment requisites of the textile industry for 1965. It determines the composition of machine stock in 1965 and, also, the extent of new building space needed to supplement the old. In prescribing the conditions in 1965, the program also defines the plan of action that must be executed by then. (4)

3. The conditions. We cannot just carry out any investment program of our choosing. There are hard factors — technical interrelations, economopolitical demands — to which we must accommodate. We have expressed these factors as conditions written in mathematical form. We consider a program "feasible" if it fulfills these conditions.

The conditions may be classified in four main groups.

The first group prescribes the external production responsibilities of the textile industry. The machine capacity of the industry must be channeled so that it will fulfill the 1965 production plan. Separate conditions govern the distribution of narrow-smooth, broad-smooth and fancy-weft textiles.

When we compiled the initial data for our calculations (mid-1960) the 1965 production plan was still not definitively set. According to the then prevailing estimate, 310.8 million square meters of raw textiles were to be produced in 1965. (We shall refer to this as "the lower production plan.") However, the suggestion was also made that production should be raised by another 50 million square meters. (This we shall later refer to as "the higher production plan.") We made parallel calculations for both cases.

The second group of conditions limited the investment sources available to the textile industry. Here we used the following:

Independent of our calculations, the Bureau of Light Industry had already prescribed the amount of the textile industry's total investment; and, within that, the sum to be spent on construction and also on machines imported from capitalist countries. (5) Whatever program our calculations recommended, none could exceed the investments, construction and hard currency of the original allowance. (6)

The third group of conditions prescribes the vertical proportions within the textile industry. We must make sure that machines engaged in pre-weaving operations serve the weaving machines without any hitch. This group of conditions actually consists of material balances expressed in mathematical form.

Some special material balances can also be enumerated here. For instance, the new automatic weaving machines cannot be served by just any kind of pre-weaving machines, but only by up-to-date ones assuring good yarn quality. This is essential to ensure that the automatic weaving machines reach their special achievement quota as assumed in our calculations. This special quality requisite was dealt with in the form of conditions.

Finally, the fourth group of conditions was the situation existing at the time of the programming, inherited from the previous period, expressing the "status quo," which naturally must be taken into consideration in determining further action.

On the one hand, the existing buildings are constants. Space demands of machines to be placed in the old buildings must not exceed the limits of existing building space.

On the other hand, existing machine stock is also a constant, about which the program should determine: should we keep the old machines, recondition, or dismantle them?

4. Optimum criteria. The number of feasible programs fulfilling the above described condition system is very great. Which among them should we consider optimal?

The one that fulfills the prescribed production plan (and, of course, the other conditions as well) at the lowest cost — this prerequisite is expressed by the program's aim, which we wish to minimize.

We have listed among the costs only those investments whose size depends on the composition of the machine stock, and generally on the investments prescribed by the program. Here, above all, belongs the total investment cost and, further, (from the operation cost sector) the wages of directly serving employees and immediate technical supervisors, the energy costs, the price of certain auxiliary materials and so on. But we can disregard those expenses which — at the volume established by the production plan, and the fixed composition of items assumed in the calculations — are independent (or nearly so) of the machine stock composition and the investment program. (Thus, for example, the bulk of direct materials costs.)

As in all investment effectiveness calculations, here too the totalling of costs appearing at different times poses a difficult problem. For this purpose we applied two formulae alternately:

One was the investment efficiency formula of the national Planning Office (the "G_n" index) which adds 20 per cent of investment costs to yearly operational costs. This can be regarded as a calculative "simple interest" of 20 per cent on the amount of the investment.

The other was the discount method. In this case we total the entire investment and operational costs of a longer, 25-year accounting period, discounting them according to the year they appear (at 10 per cent compound interest). This formula allows us to envisage that, if we build a new factory, it will require several years of investment in contrast with a simple exchange of machinery that can be undertaken in a short period. Besides, in some models we count on the fact that, in the socialist planned economy, wages systematically rise. The discount formula, totalling the costs of a longer span, makes the consideration of these periodic changes and rising wage trends easier.

In our calculation model both the aim and the conditions had a linear character. Thus we had to engage in so-called linear programming.

The dimensions of programming may be characterized by the number of the conditions and variants (alternate economic functions). In the case of the experimental programming at the Goldberger factory, we used models with four conditions and eleven variants, while now we had been working on the basis of a considerably larger model embracing twenty-four conditions and fifty variants. (7)

Practical Deductions

Our research has had a dual aim. First, to evolve practical, applicable deductions in connection with the present investment problems of the textile industry. Second, to gain overall results in the mathematical programming of investments.

We first introduced deductions of a practical character. While doing this, we made comparisons with the originally worked out investment plan set up earlier and independent of our calculations by the usual planning methods. (The task of our research was precisely to make a type of control calculation with the methods of mathematical programming.) In what follows I will call the previously made estimates the "original program," in contrast to the "proposed program" based on our calculations. (8)

1. The original program sought to condemn totally a relatively small portion of the old machine stock. At the same time, it envisaged significant construction. By contrast, the proposed program stresses modernizing existing plants and seeks to hold construction to a minimum. (9) The table below indicates the differences.

The proposed program (for the lower production plan) uses essentially the whole investment framework, but within it leaves the building budget almost completely unused, spending that sum on machinery instead. It is worth noting that even from the standpoint of textile industry costs alone, it is advisable to hold building construction to a minimum, not to mention that this is desirable in any case since building construction faces shortages in the national economy.

Creating a brand new modern plant may be more engaging work than modernizing an older plant, but outmoded old plants cannot be borne with as a permanent handicap. The technological staff of the textile industry must use all its resourcefulness and know-how to utilize every square meter of the old plants to the best advantage.

2. The calculations unequivocally proved the advantage of speeding up the old smooth weaving machines by minor alterations.

3. It is advisable to take a significant step toward automation of the textile industry. Radical replacement of outdated pre-weaving machine stock proved especially economical. Automation and the proportion of machine replacements (particularly in pre-weaving) could be much greater than in the original program. This is shown in the

Construction Requirements

Item	Original program for the lower production plan	Proposed program for the lower production plan	Proposed program for the higher production plan
From all net investments			
Machinery	67%	98.6%	90%
Buildings*	33%	1.4%	10%

*) Without the sums used for the renovation of buildings.

following table dealing with the lower production plan.

Relative Production Yield of Modern Machines (in %)

Item	1960 Actual Situation	1965 Original Program	1965 Proposed Program
Weaving machines	14	35	57
Sizing machines	0	57	100
Upcasting machines	0	53	100
Weft spooling machines	0	54	60
Cross spooling machines	36	66	100

How can this greater machine-investment program be financed? (We have stressed that the originally planned investment sum cannot be exceeded.)

As previously mentioned, a great portion of the sum originally earmarked for building should be used for machinery.

Machinery replacement obviates general repair. The old machines will not, the new machines do not, need it. To consider this possibility of savings, we calculate with a gross investment budget. This comprises not only the net budget but also the modernization budget, including general repair. It is characteristic that the original program earmarked only 73 per cent of the gross machinery investment for the purchase of new machines and 27 per cent for general repair. In contrast, the proposed program uses 91 per cent of the gross machinery investment for new machines and only 9 per cent for general repair. (10)

4. The original program did not adequately assure proportionateness among the vertically linked sectors. So, for example, the originally planned capacity of certain pre-weaving sectors seems exaggerated: the capacity surplus is much more than the technologically warranted reserve.

Similarly, there is a certain measure of contradiction in the original program between the proposed ratio of articles (the proportion of the narrow-smooth, broad-smooth and fancy-weft articles) and the planned machine capacity (the proportion of the narrow-smooth, broad-smooth and fancy-weft machines).

The proposed program is free of these drawbacks, since the conditions of the model had already taken care of maintaining the necessary proportions.

5. In the textile industry, as in all areas, the advantages and disadvantages of the various types of machines are debated. Programming results enable us to take a stand on a few debatable questions. So, for instance, it is clear that in our present circumstances weaving machines without bobbins are not efficient. Though most productive, they are the most expensive type. On the other hand, in cross-weaving machines, use of the more expensive but more productive type proved more efficient.

6. The proposed program ultimately results in a 15 per cent saving by comparison with the original program. (11)

In the sphere of practical deductions we must add one more general remark.

The final results of mathematical programming — by their very nature — delineate the problems in minute detail (e.g., 4,943 weaving machines should be dismantled, and so on). However, we must caution those charged with the practical implementation of calculated results that the recommendations of the program are not to be taken literally; this would be impractical. Like all econo-mo-mathematical models, ours too conveys a simplification of reality. It must disregard certain complex factors and neglect some interrelationships. In addition, some of the data used is not precise, being unavoidably based on more or less uncertain estimates. On the basis of the programming model, we can decide on the "strategy" of investment and technological development policies. The elaboration of "tactics," the planning of operational details, can only come later, after the practical application of programming results. This, incidentally, also means that where relationships existing in reality but bypassed in the model require it, the program should be revised in greater or lesser degree in accordance with the electronic computer's prescription.

"Sensitivity" Research

The results of the calculations are significantly influenced by data that cannot be considered unequivocally defined as to size, that is more or less indefinite — such as, for example, interest rates, foreign exchange rates, the achievement quotient of certain machines, etc.

To develop a practical viewpoint we had to examine the "sensitivity" of the optimal program to size modifications of equivocal data. I call the calculations serving to clarify this question "sensitivity research."

We engaged in many types of such research. (12) The most common method is to set up parallel programmings based on various models with differing inceptive data (for instance, higher and lower interest rates). We compare the optimal programs — possibly differing from each other — thus gained. (13)

The research showed that not all programming elements are uniformly sensitive, but there are stable results. For example, no matter what actual interest or foreign exchange rates we used, reconstruction of the preparatory phases proved advantageous.

We can also regard as a stable result the fact that all programs consistently bypassed certain alternatives. Economic leadership can be helped not only by positive proposals, but also by proof that it is quite impracticable to realize certain already argued ideas. So, for instance, in the light of sensitivity research, we may firmly reject the idea that old weaving machines can be modernized through imported "superimposed automatic devices" — although this idea has cropped up more than once in the textile industry.

Besides "stable" (definitely good or bad) alternatives, there are also "equivocal" ones. They sometimes do, or do not, get into the optimal program, depending on the assumptions used in our model. Thus it is important to know (and sensitivity research shows just this) the factors and economic relationships to whose changes they most sensitively react. Let us go into some of these sensitive points in the program.

1. The program's interest sensitivity. We made many kinds of calculations to determine the sensitivity of the program to variations of the mathematical formulae serving to express the time factor (simple interest or discounting), as well as to the numerical size of the interest rate. Among these calculations the following is especially illuminating:

We applied an aim similar to that used in the Central Planning Bureau formula. That is, to the yearly operating cost we added a certain portion of the interest cost, but we did not decide in advance what per cent this portion, this calculative simple interest rate, should be. Instead, we examined how the optimal program would be modified if we "projected" the rate through the whole range, from 3 to 30 per cent. This procedure is called parametric programming. In that event we regard the optimal program as a function of a certain parameter — in this case the interest rate.

It turned out that through changing the interest rate we actually arrived at three main program types. (14) One we might call the program of "ultra-radical" technological development. This obtains when the simple interest rate is lower than 6 to 7 per cent. Here the program would dismantle even old machines whose speeding up — as we noted — achieves good results; it proposes the purchase of the most productive, but most expensive, automatic machinery, and so on. All this demands vast investment, many times that of the original budget. The other extreme is the program of absolute technological conservatism, which prescribes that we retain all the old machines and does not recommend the replacement of a single one. This program prevails when the interest rate is above 15 per cent.

For our part — after detailed considerations which cannot be dealt with here — we took the 10 to 15 per cent interest rate range as a basis in arriving at a practical assessment. The resulting program falls between the two extremes. Although in modest degree, it definitely leads in the direction of technological development, suggesting partial replacement of machinery and partial automation (even if by using not the most productive but a relatively less expensive type).

It would be a mistake to draw far-reaching conclusions from the programming of the textile industry concerning the size of the investment utilization coefficient applied in the calculative interest rate and investment efficiency formulae. This requires economo-theoretical and economo-political ratiocination whose elaboration would extend beyond the bounds of this article. Still the phenomena here described merit attention: the usual 20 per cent interest rate decisively favors technological conservatism as against increased productivity. (Let us just consider that the proposed program — in the direction of technological development — calculated on the basis of a less than 20 per cent interest rate, can "fit" into the investment budget set up according to the cost requirements of the original investment plan based on the usual efficiency calculations.)

2. The effect of foreign exchange rates. To my mind, in investment efficiency calculations we must use exchange rates expressing what domestic outlay (in forints) produces one ruble or one dollar via exports, or, in other words, creates the foreign exchange with which we purchase the machinery imported for investment purposes.

Diverse calculations show that the exchange rates appearing at present in the mandatory, prescribed investment effectiveness indices are at odds with this principle. For this reason we made parallel calculations: one with the official rates, the other with a "corrected exchange rate that, in our view, was more in line with the above requirements. A comparison of the two types of programming shows that the official rates favor certain import relationships unreasonably, at the expense of others, while the corrected exchange rates iron out this disproportion.

It would be useful for us to examine — on the basis of far-reaching and inclusive calculations — what conversion keys and foreign exchange rates it would be advisable to use in investment effectiveness calculations.

3. The unreliability of certain achievement indices. There has been a debate among engineers and economists in the textile industry on whether it would be more practical to use spool-changing or weft-changing automation. The latter is technologically more advanced but the former is far cheaper.

In the optimal program, the cheaper type was used. By way of control, we calculated the critical technological achievement capacity (casting per machine hour), upon reaching which the weft-changing type would replace the spool-changing type in the optimal program. At this critical achievement capacity the consistent savings in operational costs would counterbalance the higher purchase price. It evolved that it is feasible to attain the critical achievement capacity.

However, to reach a final conclusion it is necessary to observe the two machines under similar conditions, in normal operation, over a longer stretch of time. Then, once we possess the adequate, representative data, the controversial question can be settled. Thus for now — instead of a final conclusion — we have had to propose the organization of comparative observation.

This problem reminds us that we should take greater care — before making final purchasing decisions — to arrange for prior observation of the characteristic operational and technological specifications of new machine types. In the event of a choice between several machine types, the comparison of prior observation data should be assured. Unfortunately, the authorities who decide on investments often fail to organize prior observation — which excessively increases the uncertainty of investment effectiveness calculations.

Sector Optimum and National Economy Optimum

There are by now two tested, deep-rooted, routine planning techniques: the balance method and economic effectiveness calculations. The balance method is designed to assure the necessary proportionality, while the effectiveness calculations rank the different alternatives, selecting the more advantageous ones.

Programming performed in industry branch dimensions involves the organic linking of these two "traditional" procedures: the balance method and economic effectiveness calculations. With some simplification we might say that in our programming model the conditional equations assume the function of balances, assure proportionality within the branches, the prescribed proportionate relations with the consuming sectors of the national economy and the available energy sources; while the graphic aim serves the function of effectiveness calculations, influencing choices from the standpoint of effectiveness.

But all this — we stress it emphatically — occurred within the frame of a single sector in the case of the textile industry calculations. The sector optimum worked out for the industry branch is not necessarily the same as the optimum for the national economy.

Concretely, we determined the optimal program for the textile industry in instances where the national economy plan had already prescribed how many million square meters of textiles were to be produced, how much money was available for investments, and so on. Our calculations reached the national economy optimum only if these figures were correct. In the course of our programming we did not investigate their accuracy.

Consequently the mathematical programming of a specific branch's expansion does not render superfluous the use of other planning methods. This is one method of planning that, however, does not replace the research, for instance, that compares the economic effects of expansion plans in different branches and various industrial sectors. Naturally, it does not replace but, on the contrary, regards as its own starting point the planning accomplished within the dimensions of the national economy: the centralized definition of balances in the national economy and proportions between the industry branches, the planned assessment of the total energy sources of the national economy, etc. (15)

At the same time, programming at the branch level can affect the planning in the national economy: it can usefully aid it. So, for instance, we examined the optimal program in the event of a lower and a higher production plan. We did not seek to reach any decision on the advisability of expanding the plan, but we did determine the amount of minimum excess cost at which an expansion of the plan by 50 million square meters could be realized. Or another example. We took as our starting point a hard-currency budget originally approved by higher authorities, and then calculated also whether savings could be made by its dissolution.

The value of the programming (more exactly, the programming series executed on the basis of diverse starting points) lies in the fact that, among other things, it shows the results of certain economo-political decisions. Moreover, it throws light not only on a single random consequence (for instance, what currency-requirement surplus will derive from the dissolution of the hard-currency budget) but reveals the whole system of consequences, showing how a given decision affects investment requisites, construction requisites, the composition of machine stocks, the structure of machine imports, and so on. A broad knowledge of effects can be an important support for high level economic leadership in making economic decisions.

Further Paths of Research

Recently the economic leadership has had more and more urgent and frequent need of applying mathematical methods to determine the effectiveness of investments. (16) The textile industry calculations comprise the first domestic attempt to satisfy this need. And for this reason the methodology described here is still not completely elaborated; it needs supplementation and perfecting.

Utilizing our experience in the textile industry, research in programming investment and expansion is now underway in two other areas, two other industry branches. In the course of these calculations we are striving to make headway in the further development of investment programming methodology:

1. We will also use a non-linear (so-called convex) programming model. This makes possible the observation of the relative savings and cost regression characteristic of larger plants.

2. The effect of investments in a given sector interacts with other sectors, other branches, of the national economy. Take, for example, intermeshing investments, the creation of capacity to supply material to new establishments under our scrutiny. For the assessment of spreading, indirect effects, connected results and interacting investments, we use input-output analysis.

3. We should like to develop further the technique of handling unavoidably uncertain data used in investment calculations. As we know, with different alternatives the degree of uncertainty is not the same: the realization of each involves greater or less uncertainty and risk. We intend to use methodology that can sense differences in the degree of uncertainty.

Here I have referred only briefly to a few new traits of the programming still underway. A more detailed account will be timely when the numerical calculations are completed.

In the further development of programming methods the best and most reliable road to our goal is practical testing. While solving a concrete calculation problem we must always compromise between the theoretically "ideal" construction and actual possibilities (in the light of insufficient available data, computer limitations, urgent deadlines).

Still, I think that such an incomplete, but numerically solved, calculation arriving at practical proposals may be worth more than the most satisfactory complete model existing only on paper. The inspiration of theoretical advance is the practical solution of problems.

APPENDIX

Description of the Calculation Model

In Model No. 1 there are fifty variants; among them forty three are worthwhile economic measures. The itemized variants follow:

1. The continued operation of the old, poorer, narrow-smooth weaving machines. (We divided the old narrow-smooth machine stock into two groups on the basis of technological specifications: the categories of less productive, "poorer," and more productive, "better" machines. We also made a similar division between the old cross-spooling and weft-spooling machine stock.)

2. Dismantling of old, poorer, narrow-smooth machines.

3. Continued operation of old, better, narrow-smooth machines.

4. Dismantling of old, better, narrow-smooth machines.

5. Speeding up of old, better, narrow-smooth machines. (This involves minor remodeling that makes a significant increase in revolutions possible. Among technological experts, it is a debatable question what portion of the old machine stock can be speeded up. We may even have been too cautious in restricting the sector of machines that can be speeded up to the "better" category.)

6. Automation of old, better, narrow-smooth machines with imported equipment. (In this case the machines are modernized by installing so-called "superimposed automation.")

7. Installing new, spool-changing, narrow-smooth, automatic machines in old plants. (This is the cheapest but least productive type.)

8. Installing new, spool-changing, narrow-smooth, automatic machines in new plants. Adequate sizing, upcasting and weft-spooling capacity is linked to machines placed in new plants and buildings — here, and later in similar variants. Thus it is not only a question of purchasing and installing weaving machines but, to this, a "pre-weaving aggregate" is linked, along with the creation of the building space in which the weaving machines and pre-weaving aggregates are placed. We put all this — in this form — into the model because, in the case of a new plant, the required sizing, upcasting and weft-spooling capacity must certainly also be available on the spot. By contrast, cross-spooled yarn can be shipped and, for this reason, the installation of cross-spooling machines does not appear in the "aggregate."

9. Installation of new, spool-changing narrow-smooth, automatic weaving machines in old plants. (This is more productive than the previous type but more expensive.)

10. Installation of new, spool-changing, narrow-smooth, automatic weaving machines and the connected pre-weaving aggregate in new plants.

11. Installation of new, bobbinless, narrow-smooth, automatic weaving machines in old plants. (This type is more productive, but more expensive than either of the previous types.)

12. Installation of new, bobbinless, narrow-smooth, automatic weaving machines and connected pre-weaving aggregates in new plants.

13. Continued operation of old, broad-smooth weaving machines.

14. Dismantling of old, broad-smooth weaving machines.

15. Installation of new, broad-smooth weaving machines in old plants.

16. Installation of new, broad-smooth weaving machines and connected pre-weaving aggregates in new plants.

17. Continued use of old, fancy-weft weaving machines.

18. Dismantling of old, fancy-weft weaving machines.

19. Installation of new, cheaper, automatic fancy-weft weaving machines in old plants.

20. Installation of new, cheaper, automatic fancy-weft machines and connected pre-weaving aggregates in new plants.

21. Installation in old plants of new, automatic fancy-weft weaving machines that are more expensive but more productive than the previous type.

22. Installation of new, more expensive, automatic fancy-weft weaving machines and connected pre-weaving aggregates in new plants.

23. Continued operation of old sizing machines.

24. Dismantling of old sizing machines.

25. Modernizing of old sizing machines. (The

cabinets are rebuilt, automatic control devices installed, etc.)

26. Installation of new automatic sizing machines. (In installing the new sizing machines, as well as the new upcasting and weft-spooling machines, it is always a question of placing them in old plants. The possibility of installing new sizing, upcasting or weft-spooling machines in new plants is already included in the pre-weaving aggregates linked with the weaving machines placed in new plants.)

27. Continued operation of old upcasting machines.

28. Dismantling of old upcasting machines.

29. Installation of new, cheaper, upcasting machines.

30. Installation of new upcasting machines — more expensive, but more productive, than the previous type.

31. Continued operation of old, poorer weft-spooling machines.

32. Dismantling of old, poorer weft-spooling machines.

33. Continued operation of old, better weft-spooling machines.

34. Dismantling of old, better weft-spooling machines.

35. Installation of new, automatic weft-spooling machines.

36. Continued operation of old, poorer cross-spooling machines.

37. Dismantling of old, poorer cross-spooling machines.

38. Continued operation of old, better cross-spooling machines.

39. Dismantling of old, better cross-spooling machines.

40. Installation of new, relatively cheaper, automatic cross-spooling machines in old plants.

41. Installation of new, relatively cheaper, automatic cross-spooling machines in new plants.

42. Installation in old plants of new, automatic cross-spooling machines, more expensive but more productive than the previous type.

43. Installation of new, more expensive cross-spooling machines in new plants.

Besides the listed 43 variants there are also 7 so-called auxiliary variants in the model. These are needed so that we may, by their inclusion, equalize the inequalities on the basis of their economic content.

The model contains 24 conditions. We can summarize the system of conditional equations in the following equation:

(1) $Ax = b$,

where A = the matrix of the coefficients a_{ik} ($i = 1, 2, \ldots, 24$; $k = 1, 2, \ldots, 50$) occurring in the conditions,

x = the vector of the program,

b = the vector of the constants appearing at the right of the conditional equations.

To illustrate the structure of the condition-system we show in the appended table matrix A and vector b appearing in Model No. 1. Since the table serves to illustrate the construction of conditions, we use the following symbols instead of numerical values:

The "a" symbol appears where, in the original numerical matrix, there are numbers other than "0" and "1." The number, 1, appears in instances where it also appeared in the original matrix. Finally, we left the table blank where 0 appeared in the numerical matrix. (See table on page 53.)

Similarly in vector $\underline{b}$ we use the symbol "b" where numbers other than 0 appeared in the numerical vector.

The table on page 53 serves to show the content of conditions.

A program is optimal when it fulfills the condition system described in formula (1) and where the value of $C(x)$

$$(2) \qquad C(x) = \sum_{k=1}^{50} c_k x_k$$

should be minimized.

The content, and consequently the numerical value, of cost factor c_k varies depending on which type of graphic aim we apply.

In Model No. 1 we defined the cost factors $c_k^{(1)}$ as follows:

$$(3) \quad c_k^{(1)} = \sum_{t=1}^{25} \frac{q_k(t)}{(1+r)^t} + \frac{p_k(t)\,(1+z)^t}{(1+r)^t} \qquad (k=1, 2, \ldots, 50)$$

where $q_k(t)$ = a non-wage-type cost appearing in connection with one unit of the k variant in year t,

$p_k(t)$ = a wage-type cost appearing in connection with one unit of the k variant in year t,

r = the interest rate,

z = rate of yearly wage increase.

We define as follows the cost factor $c_k^{(4)}$ applied in Model No. 4:

$$(4) \qquad c_k^{(4)} = b_k \lambda + d_k \qquad (k=1, 2, \ldots, 50)$$

where b_k = the single investment cost, The Conditional Equation System on Matrix A and Vector b

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	a		a		a	a	a	a	a	a	a	a													
2													a		a	a									
3																	a		a	a	a	a			
4	-a		-a		-a	-a	-a		-a		-a		-a		-a		-a		-a		-a		a		a
5	-a		-a		-a	-a	-a		-a		-a		-a		-a		-a		-a		-a				
6	-a		-a		-a	-a	-a		-a				-a		-a		-a		-a		-a				
7	-a		-a		-a	-a	-a	-a	-a	-a	-a	-a	-a		-a		-a		-a	-a	-a	-a			
8						a	a		a		a				a				a		a				-a
9						a	a		a		a				a				a		a				
10						a	a		a						a				a		a				
11	a	-a	a	-a	a	a	a	a	a	a	a	a	a	-a	a	a	a	-a	a	a	a	a	a	-a	a
12						a		a		a	a	a				a				a	a	a			
13	a		a		a	a	a		a		a				a		a		a		a		a		a
14								a		a		a				a				a		a			
15	1	1																							
16			1	1	1	1																			
17													1	1											
18																1	1								
19																							1	1	1
20																									
21																									
22																									
23																									
24																									

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	b
1																										b
2																										b
3																										b
4	a																									b
5		a		a	a																					b
6						a		a		a																b
7											a		a		a	a	a	a								b
8	-a																		1							o
9				-a	-a															1						o
10										-a											1					o
11	a	a	-a	a	a	a	-a	a	-a	a	a	-a	a	-a	a	a	a	a				1				b
12					a					a													1			b
13	a	a		a	a	a		a		a	a		a		a		a							1		b
14																a		a							1	b
15																										b
16																										b
17																										b
18																										b
19																										b
20		1	1																							b
21						1	1																			b
22								1	1																	b
23											1	1														b
24													1	1												b

The Economic Content of the Conditions

Condition Number	Content of Conditions	Content of Coefficient a_{ik}	Content of Constant b_i appearing on the righthand side
1	plan of narrow-weave mill	yearly potential of machine	yearly production plan
2	plan of broad-weave mill	yearly potential of machine	yearly production plan
3	plan of fancy-weft mill	yearly potential of machine	yearly production plan
4	general sizing balance	with positive sign: yearly potential of sizing machine; with negative sign: yearly yarn requirement of weaving machine	requirements of machines not included in the plan
5	general upcasting balance	analogous to condition No. 4	requirements of machines not included in the plan
6	general weft-spooling balance	analogous to condition No. 4	requirements of machines not included in the plan
7	general cross-spooling balance	analogous to condition No. 4	requirements of machines not included in plan, and of sectors outside the industry
8	special sizing balance	with positive sign: yearly potential of modern sizing machine; with negative sign: yearly requirements of automatic weaving machine	
9	special upcasting balance	analogous to condition No. 8	
10	speciaı weft-spooling balance	analogous to condition No. 8	
11	gross investment budget	with positive sign: investment cost requisites; with negative sign: income from dismantling operations	1961-65 budget
12	hard currency budget	hard currency requirements of machine imports	1961-65 budget
13	old plant space	space requirements of machines placed in old plants	total old plant space
14	building budget	space requirements of machines placed in new plants	total new plant space allowed by the building budget
15	distribution of the old, poorer narrow-smooth weaving machine stock		stock of old, poorer narrow-smooth weaving machines
16-24	distribution of other groups of old machine stock		analogous to condition No. 15

d_k = the yearly operational cost,

λ = the (simple) interest rate.

In the following, we consider briefly how the 17 types of models used in the calculations series differ from each other. The more important differences:

Some models include the narrow, the broad, as well as the fancy weft machines ("big models"); others only the narrow machines, and, consequently, handle fewer alternatives ("small models"). The use of compact, small models was necessary to enable us to make certain supplementary calculations without using electronic computers.

In some models we applied discounting formulae; in others, simple interest formulae.

In some models we took into account the trend toward wage increases; in others, not.

In some models we calculated using the official foreign exchange rates; in others, the corrected ones.

In some models we took the lower production plan; in others, the higher production plan.

In some models we viewed the investment sources as limited; in others as unlimited.

In the "small models" we solved the problem of linear programming with the usual simplex procedure. For the programming executed by the "big models" we used a somewhat modified variant of the simplex procedure, easily applicable to the limited memory capacity of the electronic computer.

By utilizing various models we performed a number of "sensitivity researches." One aspect in the classification of such research involves the question of where in the model the change occurs whose effects we wish to observe:

1. What is the effect if we modify cost factor c appearing in the graphic aim? (This happens, for instance, in case of an interest rate change.)

2. What is the effect if we modify vector b of the constants appearing on the right side of the conditional equations? (This happens, for instance, in the event of a change in the production plan.)

3. What is the effect if we put a modified a_k columnal vector in the place of one of the original a_k columnal vectors of matrix A, and simultaneously, if, in place of cost factor c_k appearing in the graphic aim, we also use another c_k cost factor? (This happens when we modify the technical-economic indices of some alternative — for example, the assumed potential of a machine.)

4. What is the effect if we leave out certain lines of Matrix A? (This happens when we cancel the limitations of a limited energy source.)

In the first three problem types of the above enumeration, two kinds of solutions are provided for the sake of completeness:

a) So-called parametric programming provides a more complete answer. Here we regard the optimal program as the function of some g parameter. The problem is to define the value of function x (g) with respect to all possible values of g (or at least with respect to all the values of g within a fixed interval). We employed this procedure in one of our researches relating to interest rates.

b) We do not consider all possible values of the mentioned problematic parameter, but do the programming separately for only a few previously selected values. (We applied this approach, for instance, to the production plan, or foreign exchange rates.) This obviously gives a less complete response to the effects of change in the problematic parameter than the method introduced under a).

Further details of programming and related technological, statistical and mathematical problems are dealt with in a detailed study. ["A pamutszövőipar optimális beruházási programjának meghatározása," Textilipari Kutató Közleményei, 1960, mimeographed.]

Footnotes

1) The results of the experimental programming were reviewed in my article that appeared in the June 1960 issue of Közgazdasági Szemle, "A műszaki fejlesztés és a beruházások gazdaságosságának számitása."

In this study I aim to avoid unnecessary repetition by comparison to last year's article. For that reason I do not repeat the definition of concepts already clarified there.

2) Collaborating colleagues in the data collection and numerical calculations of the industry branch programming were: Ferenc Korányi (Kistex), Zoltán Papp (Goldberger), Dr. János Pécsi (Goldberger), László Szabó (Textilipari Kutató Intézet) and Péter Wellisch (Textilipari Kutató Intézet).

3) In our country this was the first general linear programming mission of substantial dimensions to be solved by electronic computers. The numerical calculations were supervised by Tamás Frey

(Számitástechnikai Központ).

4) The calculation did not deal with the pace of the operational plan.

5) We did not regard machine imports from socialist countries as specifically limited, but assumed that the entire investment budget could be freely divided between domestic investments and machine imports originating in the socialist countries.

6) We calculated with a so-called "gross investment budget." Besides the net investment budget, this included the machine stock renewal budget. (We did not here include the building construction budget.)

7) The number of variants also includes the auxiliary variants.

8) The "original program" sought to assure fulfillment of the lower, 310.8 million square meter, production plan. So, in comparisons, we always contrast it with the proposed program that is destined to fulfill the same production plan. Insofar as we mention calculations assuming the higher plan, raised by a 50 million square meter output, we make special reference to this.

9) Our calculations — in the case of the textile industry — fully justified the directive already stressed in many official documents in the Soviet Union and in Hungary. So, for instance, theses dealing with the directives of the Second Five-Year Plan emphasized: "During the years of the Second Five-Year Plan we must increasingly strive to achieve production expansion not by building new plants, but by optimum utilization (and, if need be, expansion) of existing ones, through more efficient use of extant capacity, and by modernizing production procedures."

10) The calculation of the gross investment budget is naturally not tied to the application of mathematical programming. It would also be worth systematically weighing this regrouping possibility in drawing up the investment plans by customary methods.

11) The value of the graphic aim in the case of the original program is 5,946 million forints, and, in the case of the suggested program based on mathematical programming, 5,048 million forints. Both amounts refer to the investment costs in the weaving mills of the textile industry during the period of the Second Five-Year Plan, and the discounted operational costs incurred during the succeeding 25 years.

12) In the Appendix I describe the classification of the sensitivity researches from the viewpoint of the mathematical nature of the program.

13) Within the framework of our calculation series we arrived at several optimal programs differing from each other. They are all optimal in the sense that, within the frame of the given model, along with the given numerical values, each program minimizes the graphic aim. Through comparison and economic evaluation of these diverse "optimal" programs, we ascertained which could be regarded as most favorable. The first part of the article describes conclusions derived in such fashion by deduction from the sensitivity researches.

14) In this calculation we did not limit available investment sources.

15) It is another question whether these mathematical programming methods could be used in national economy dimensions; their utilization could perfect present planning techniques.

16) The need appears not only in our own country, but also in other socialist lands. So, for example, great stress was laid on this task at the conference called by the Soviet Academy of Sciences for the practical application of mathematical methods. (See Voprosy ekonomiki, 1960, No. 8.) A resolution was passed stating: "Mathematical methods determining the economic effectiveness of investments will be worked out by a series of government committees and institutions..."

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